Peak-to-average power ratio reduction in all-optical orthogonal frequency division multiplexing system using rotated constellation approach

Jassim K. Hmood, Kamarul A. Noordin, Hamzah Arof, Sulaiman W. Harun

Department of Electrical Engineering, Faculty of Engineering, University of Malaya, 50603 Kuala Lumpur, Malaysia
Laser and Optoelectronic Department, University of Technology, 10066 Baghdad, Iraq
Photonics Research Center, University of Malaya, 50603 Kuala Lumpur, Malaysia

In this paper, a new approach for reducing peak-to-average power ratio (PAPR) based on modulated half subcarriers in all-optical OFDM systems with rotated QAM constellation is presented. To reduce the PAPR, the odd subcarriers are modulated with rotated QAM constellation, while the even subcarriers are modulated with standard QAM constellation. The impact of the rotation angle on the PAPR is mathematically modeled. The effect of PAPR reduction on the system performance is investigated by simulating the all-optical OFDM system, which uses optical coupler-based inverse fast Fourier transform (IFFT)/fast Fourier transform (FFT). The all-optical system is numerically demonstrated with 29 subcarriers. Each subcarrier is modulated by a QAM modulator at a symbol rate of 25 Gsymbol/s. The results reveal that PAPR is reduced with increasing the angle of rotation. The PAPR reduction can reach about 0.8 dB when the complementary cumulative distribution function (CCDF) is $10^{-3}$. Furthermore, both the nonlinear phase noise and the optical signal-to-noise ratio (OSNR) of the system are improved in comparison with the original all-optical OFDM without PAPR reduction.

1. Introduction

All-optical orthogonal frequency division multiplexing (OFDM) system has drawn great attention in recent years due to its potential applications in high-bit-rate transmission systems [1]. Furthermore, this system provides a better tolerance to chromatic and polarization-mode dispersions compared to the conventional systems [2–5]. For instance, the all-optical OFDM system eliminates the requirement of electronics signal processing and thus it is much more feasible for high bit rate transmission system [1,6,7]. However, all-optical OFDM system highly suffers from phase noise which introduces phase rotation for each subcarrier and thus destroys orthogonality of subcarriers [8]. The phase noise is mainly induced from fiber nonlinear effects such as cross-phase modulation (XPM) and four-wave mixing (FWM) [9]. This is evident when adding a number of subcarriers in the time domain for high power transmission signals [10]. The combined signals induce the fiber nonlinear effects and degrades the system performance [11,12]. Therefore, many approaches have been proposed and reported to mitigate fiber nonlinear impairment during transmission of signals in optical OFDM systems where high peak-to-average power ratio (PAPR) reduction is the popular approach.

In both wireless and conventional optical OFDM systems, PAPR reduction is realized in the electrical domain. Various techniques have been developed to reduce PAPR in optical OFDM systems such as amplitude clipping and filtering [13,14]. Although the implementation of clipping technique is simple and less complex, the clipping processes produce a distortion in the optical signal hence increasing the bit error rate (BER). There are also other techniques such as selected mapping (SLM) and partial transmit sequence (PTS) methods which are considered as effective for reducing PAPR in conventional optical OFDM systems [12,15]. However, these methods involve a high computational complexity. Furthermore, the constant envelope of the electrical OFDM waveform has also been adopted to improve the tolerance of MZM nonlinearities and to relax the requirements of digital-to-analog and analog-to-digital converter (DAC/ADC) [16,17]. Indeed, few investigations have been reported on the PAPR reduction techniques in all-optical OFDM systems [18]. They focus on the all-optical OFDM systems, which employ intensity modulation rather than phase
modulation. Phase pre-emphases method has been proposed to reduce PAPR in all-optical OFDM systems [19].

In this paper, we propose a simple technique to reduce PAPR based on rotated constellation in coherent all-optical OFDM system. In this approach, the subcarriers are divided into odd and even subsets. Then the constellation of odd subcarriers is rotated counter clockwise while the constellation of even subcarriers is remained without rotation. The impact of the rotation angle on the PAPR is mathematically modeled. Then, the resulting PAPR reduction on the total phase noise in all-optical OFDM systems is mathematically modeled and numerically investigated. The simulation results show that the proposed technique provides 0.8 dB PAPR reduction with a better nonlinear impairment tolerance in all-optical OFDM system that employs 29 subcarriers and symbol rate of 25 Gsymbol/s.

The rest of the paper is organized as follows. Section 2 describes the PAPR reduction principle. The effect of rotated constellation method on fiber nonlinearity is discussed in Section 3. The proposed all-optical OFDM system setup is presented in Section 4. The analytical and simulation results are presented in Section 5, where the impacts of rotation angle of constellation on the PAPR and variance of the total phase noise are studied. The validation of our analytical model using simulation results of our systems is given in the same section as well. Finally, a conclusion is drawn in Section 6.

2. PAPR reduction principle

In this section, a new approach to mitigate fiber nonlinear impairment by reducing PAPR is mathematically explained. First, the subcarriers are divided into odd and even subsets. Then, at QAM modulators, the original constellation of odd subcarriers is rotated with an angle of $\theta$ (clockwise) while the constellation of even subcarriers is determined by the standard 4QAM constellation as shown in Fig. 1. This approach is suitable for both conventional optical OFDM and all-optical OFDM systems where the constellation is realized in electrical domain. The output of the OFDM transmitter ($u(t)$) is given by

$$u(t) = \sum_{k=-(N-1)/2}^{(N-1)/2} u_k(t) \exp(j\theta) \exp(j\omega_k t) + \sum_{k=-(N-1)/2}^{(N-1)/2} u_k(t) \exp(j\omega_k t),$$

where $N$ represents the total number of subcarriers (assumed odd without loss of generality), $e \in \{-N-1, -N-3, \ldots, N-1, N+1\}$ and $\omega = \{\omega_0, \omega_2, \omega_4, \ldots\}$ represent odd and even numbers, respectively, $\theta$, $0 \leq \theta \leq \pi/4$ is the angle of rotation, $\omega_k = 2\pi k/T_s$ is the frequency offset from the reference optical carrier and $T_s$ is defined as OFDM symbol time. Here, $u_k(z,t)$ represents the normalized slowly varying field envelope of $k$th subcarrier and it is defined as

$$u_k(t) = \sqrt{P} A_k \text{rect}(t-kT_s).$$

where, $P$ is the optical power of single subcarrier, $A_k = a_k + jb_k$ is a complex number and is determined by the standard 4QAM constellation and

$$\text{rect}(t) = \begin{cases} 1 & \text{if } 0 \leq t \leq 1, \\ 0 & \text{otherwise}. \end{cases}$$

By substituting (2) in (1), the optical field the OFDM signal can expressed as

$$u(t) = \sqrt{P} \exp(j\theta) \sum_{k=-(N-1)/2}^{(N-1)/2} A_k \text{rect}(t-kT_s) \exp(j\omega_k t) + \sum_{k=-(N-1)/2}^{(N-1)/2} A_k \text{rect}(t-kT_s) \exp(j\omega_k t).$$

The maximum optical field can be obtained when all subcarriers are coherently combined. To achieve this, all subcarriers should be modulated with the same QAM symbol, making the summations of magnitudes of odd and even subcarriers equal to half summation of magnitude of all subcarriers and the angle between them is equal to the rotating angle. Then, the magnitude of optical field of OFDM signal can be written as

$$|u(t)| = \sqrt{P} \left| \frac{1 + \exp(j\theta)}{2} \right| \sum_{k=-(N-1)/2}^{(N-1)/2} |A_k|.$$ 

By doing some algebra, the magnitude of optical field can be expressed as

$$|u(t)| = \sqrt{P} \cos \left( \frac{\theta}{2} \right) \sum_{k=-(N-1)/2}^{(N-1)/2} |A_k|

= \sum_{k=-(N-1)/2}^{(N-1)/2} \sqrt{P} \cos \left( \frac{\theta}{2} \right) |A_k|.$$ 

From (5), it can be considered that the magnitude of optical field of $k$th subcarrier equal to

$$|u_k(t, \theta)| = \sqrt{P} \cos \left( \frac{\theta}{2} \right) |A_k|.$$ 

The PAPR of the signal, $u(t)$, is defined as the ratio of the peak of instantaneous power to the average power, and is given as [13]

$$\text{PAPR} = \frac{\max\left(|u(t)|^2\right)}{E\left[|u(t)|^2\right]},$$

where $E[\cdot]$ is the expectation operator. For 4QAM constellation, the $|A_k| = \sqrt{2}$ because $a_k = b_1 = 1$. The magnitude of optical field and the power of $k$th subcarrier after rotating the constellation can be expressed as

$$|u_k(t, \theta)| = \sqrt{P} \cos \left( \frac{\theta}{2} \right),

|u_k(t, \theta)|^2 = P \cos^2 \left( \frac{\theta}{2} \right),$$

respectively. The maximum power is occurred when power of $N$ subcarriers are coherently added and it equal to

$$\max\left(|u(t)|^2\right) = \sum_{k=-(N-1)/2}^{(N-1)/2} P \cos^2 \left( \frac{\theta}{2} \right) = PN \cos^2 \left( \frac{\theta}{2} \right).$$

![Fig. 1. Rotated 4QAM constellation.](image-url)
The average power can be expressed as

\[ E\left[ (u(t))^2 \right] = \sqrt{N} \frac{P}{\sqrt{2}} \]  

(10)

By substituting (9) and (10) in (7), the PAPR becomes

\[ \text{PAPR}(\theta) = \sqrt{2N \cos^2 \left( \frac{\theta}{2} \right)}. \]  

(11)

From (11), the relation between the PAPRs for the system with and without employing rotation constellation technique is factor \( \cos^2 \left( \frac{\theta}{2} \right) \). Therefore, the cumulative distribution function (CDF) of (PAPR < x) for proposed system can be written as

\[ \text{CDF} = \int_{0}^{x} \frac{y}{\cos^2 \left( \frac{\theta}{2} \right) \sigma^2} \exp \left( -\frac{y^2}{2\sigma^2 \cos^2 \left( \frac{\theta}{2} \right)} \right) dy \]

\[ = 1 - \exp \left( -\frac{x^2}{2\sigma^2 \cos^2 \left( \frac{\theta}{2} \right)} \right). \]

(12)

For large N OFDM symbols, the PAPR is considered as a random variable where its distribution is given by

\[ P(\text{PAPR} \leq x) = \left( 1 - \exp \left( -\frac{x^2}{2\sigma^2 \cos^2 \left( \frac{\theta}{2} \right)} \right) \right)^N. \]

(13)

The complementary cumulative distribution function (CCDF) describes PAPR statistics. It shows the probability of an OFDM signal exceeding a given PAPR, and is written as

\[ \text{CCDF} = 1 - \left( 1 - \exp \left( -\frac{x^2}{2\sigma^2 \cos^2 \left( \frac{\theta}{2} \right)} \right) \right)^N. \]

(14)

3. Effect of rotated constellation method on fiber nonlinearity

In this section, the effect of rotated constellation of odd subcarriers on the fiber nonlinearity impairments is described. The intensity of the optical signal is one of the main factors that affect the phase noise caused by XPM and FWM. By reducing the high peaks of the OFDM signal, the nonlinear distortion should decrease.

3.1. XPM phase noise

It is well known that XPM refers to the nonlinear phase shift of an optical field induced by another field with different wavelength, direction, or state of polarization. In long haul transmission system, the optical signal is commonly transmitted through multi-span optical fiber. Each span is constructed of a single mode optical fiber, a dispersion compensation fiber, and an optical amplifier. Normally, an optical amplifier is used to compensate power loss due to the fiber attenuation in each span. However, these amplifiers add a random noise to the transmitted signal where a field of amplified spontaneous emission (ASE) noise is added to each of the subcarriers. ASE noise is effectively white noise with variance \( \sigma^2 \). The nonlinear phase noise due to XPM in the present ASE noise is accumulated span-by-span [20], and it can be expressed as

\[
\phi_{\text{XPM}} = \gamma L_{\text{eff}} \left[ \sum_{i=1}^{M} \sum_{\mu=0}^{N-1} u_i \exp(j\theta) + \sum_{\mu=1}^{N} n_{\mu}(t) \right] + \gamma L_{\text{eff}} \left[ \sum_{i=1}^{M} \sum_{\mu=0}^{N-1} u_i \exp(j\theta) + \sum_{\mu=1}^{N} n_{\mu}(t) \right] \]

(15)

where \( \gamma \) is the nonlinear coefficients, \( L \) is the fiber length of each span, \( M \) number of fiber spans and \( n_{\mu}(t), \mu \in \{1, 2, \ldots, M\}, \) is the complex amplifier noise at the \( \mu \)th span and \( \mu \)th subcarrier. Here, \( L_{\text{eff}} = (1 - e^{-\alpha z})/\alpha \) and \( \alpha \) is the attenuation coefficient. By substituting (6) in (15), the phase noise variance can be written as

\[ \sigma_{\text{XPM}}^2(ML, \theta) = 2M(\gamma + 1)\gamma L_{\text{eff}} P \cos^2 \left( \frac{\theta}{2} \right) \sum_{i=1}^{M} |A_i|^2 \sigma_i^2 \]

(16)

From the last equation, it is clear that the angle of rotation \( \theta \) governs the variance of phase noise due to interaction of XPM with ASE noise.

3.2. FWM phase noise

The FWM process is a phase sensitive process where the interaction depends on the relative phases of all subcarriers and its effect can efficiently accumulate over longer distances. The FWM process adds a fluctuation to the optical field that causes a phase noise. For \( M \) fiber spans, the phase noise variance due to ASE noise and its interaction with FWM can be written as [20]

\[ \sigma_{\text{ASE}, \text{FWM}}^2(ML, \theta) = \frac{M \sigma_0^2 |u_i|^2}{|\Gamma_i|^2} \sum_{i=1}^{M} \sum_{k=1}^{N} \left( \sum_{j=1}^{i+k-1} \sum_{l=0}^{N-1} \frac{1}{|\Gamma_j| |\Gamma_j|^*} \right) \]

(17)

where \( L_{\text{FWM}}(2) = 1 - \exp \left[ -\left( \frac{\beta_2 r}{\pi \alpha_0^2 \sqrt{2}} \right) \right] \), \( \beta_2 \) is the dispersion profile and \( \Omega = \alpha_0^2 + \alpha_2^2 - \alpha_0^2 - \alpha_2^2 \).

First part of (18) represents the phase noise due to the ASE noise only while the last part represents the phase noise due to the interaction of FWM with ASE noise. For the proposed approach, the magnitude of the phase noise variance is governed by the angle between odd and even constellations. By substituting (6) in (15), the phase noise variance for the proposed system can be expressed as

\[ \sigma_{\text{proposed}, \text{FWM}}^2(ML, \theta) = \frac{2M \sigma_0^2 |u_i|^2}{|\Gamma_i|^2} \sum_{i=1}^{M} \sum_{k=1}^{N} \left( \sum_{j=1}^{i+k-1} \sum_{l=0}^{N-1} \frac{1}{|\Gamma_j| |\Gamma_j|^*} \right) \]

(18)

From last equation, the phase noise variance due to interaction of FWM with ASE noise is controlled by adjusting the angle of rotation.

4. All-optical OFDM system setup

The schematic of all-optical OFDM system setup [7], is shown in Fig. 2. The system includes three subsystems: transmission, trans-
mission link, and reception. The transmission subsystem consists of an optical frequency comb generator (OFCG), wavelength selected switch, optical QAM modulators, and an optical beam combiner. The OFCG utilizes two intensity modulator (IM) and one time delay to generate 29 subcarriers with constant frequency spacing [21]. Subsequently, the generated subcarriers are split into odd and even subcarriers. The odd subcarriers are modulated according to rotated QAM constellation, while standard QAM constellation is used to modulate the even subcarriers. The odd and even subcarriers are directly superimposed upon modulating by using beam combiner to generate OFDM signal.

The transmission link comprises multi-span fiber loops. Each span consists of 55 km standard single mode fiber (SSMF), dispersion compensating fiber (DCF) and an Erbium doped fiber amplifier (EDFA) as shown in Fig. 2. The SSMF is modeled with an attenuation coefficient of 0.2 dB/km, chromatic dispersion (CD) coefficient of 16 ps/nm/km, and fiber nonlinearity of 1.3 W⁻¹ km⁻¹. The DCF fully compensates the CD of SSMF with CD coefficient of −160 ps/nm/km. The EDFAs with a noise figure of 6 dB are employed to compensate the fiber loss and control the launching power into the SSMF and DCF.

The all-optical OFDM receiver comprises all-optical FFT (OFFT) circuit [22] and coherent QAM optical detectors. The main function of the OFFT circuit is to perform serial-to-parallel conversions in frequency domain. The OFFT circuit consists of three cascaded Mach–Zehnder Interferometers (MZIs), optical filters and electro-absorption modulators (EAMs). Each MZI has one optical time delay and one optical phase shifter as shown in Fig. 2. The time delay and phase shift of first MZI are adjusted to T₁/2 and 0 rad, respectively. The time delay of the other two subsequent parallel MZIs is set to T₁/4, while the phase shift of the upper and lower ones are set at 0 rad and π/2 rad respectively. Four demultiplexers are directly employed to split and filter the subcarriers. Then, the signals are sampled by EAM sampling gates. Afterwards, the output of the EAM is filtered by a third-order bandpass filter before it is demodulated by the coherent QAM optical detector.

5. Results and discussion

In order to investigate the validity of our approach, model analysis is carried out and the finding is compared with the result of numerical simulation generated by VPTransmissionMaker 9.0. The analytical and simulation results are obtained for 29 subcarriers. Each subcarrier is modulated with 4QAM constellation at a symbol rate of 25 Gsymbol/s and transmitted through multi-span fiber with a total length of 1100 km.

5.1. Transmitter side

Fig. 3 depicts the impact of rotation angle on the PAPR. The results are achieved for N = 29, 4QAM mapping and subcarrier power of −3 dBm. The comparison of CCDF performances for the proposed approach at θ = π/6 rad and θ = π/4 rad against the original system (without using any PAPR reduction method) is shown in Fig. 3(a). It is shown that for the original OFDM (θ = 0), in every 10⁴ OFDM frames has its PAPR greater than 8.9 dB. Using the rotated constellation technique, at θ = π/4 rad, in every 10⁴ OFDM frames has its PAPR exceeding 8.1 dB. The PAPR is improved by more than 0.8 dB for CCDF = 1 × 10⁻³ over the conventional system. The PAPR for various rotation angles is shown in Fig. 3(b). It is clear that the PAPR is reduced with increasing the rotation angle from 0 to π/4 rad. However, when the rotation angle is increased beyond the π/4, the PAPR is also increased because the constellation of odd subcarriers approaches to the constellation of even subcarriers in next quarter as shown in inset of Fig. 3(b). Therefore, the PAPR(θ) is considered a periodic function and the optimum angle is π/4. The analytical result (dashed line) indicates that the PAPR drops from 8.8 dB to 8.1 dB when the rotation angle increases from zero to π/4 rad. The simulation result (solid line) exhibits similar behavior and there exist a good agreement between the analytical and simulation results.

Fig. 4 shows the optical waveforms of all-optical OFDM signals for various rotation angles. The results are obtained for 4QAM mapping with different rotation angles but at the same power of subcarriers (P = −3 dBm). It can be observed from Fig. 4(a) that the original system (θ = 0) generates an optical OFDM signal with a maximum peak of 20.4 mW. By increasing the rotation angle θ to π/4, the maximum peaks of the transmitted optical OFDM signal are obtained at 17.2 mW as shown in Fig. 4(b). This indicates that the increment of the rotation angle decreases the signal peaks significantly. This phenomenon results from combining the odd and even subcarriers with different rotated angles. In agreement with (9), a higher rotation angle leads to reduction in peak power of the generated OFDM signal.

5.2. Receiver side

To show the mitigation results of the fiber nonlinear impairments, Fig. 5 depicts the nonlinear phase noise variance against
the power of subcarrier for the proposed system at rotation angles of $\theta = \pi/4$ rad and the original OFDM system. The results are quantified using both simulation and the analytical model for 29 subcarriers and transmission distance of 1100 km. It is found that the phase noise variance is successfully reduced when the angle of rotation is set to $\theta = \pi/4$. At low signal power, the phase noise variances of both systems are slightly different. This is because the performance of the system is governed by the optical amplifier noises at low signal power. However, with higher signal power, the difference between the phase noise variances becomes clearer. This is because the proposed system reduces the PAPR and mitigates the fiber nonlinear impairments without any additional distortion. In addition, it can be noted that the presented analytical results show good agreement with the simulation results.

Fig. 6(a) shows the influence of the angle of rotation on the BER performance of the all-optical OFDM system. Referring to (16) and (18), both XPM and FWM are reduced with increasing the rotation angle because the power of the OFDM symbol is reduced with employing proposed approach. Clearly, the BER reduces as the angle of the rotation increases. The highest BER occurs at $\theta = 0$, whereas the minimum
BER occurs at $\theta = \pi/4$. Furthermore, the OSNR is raised with increasing the angle of rotation. This indicates that the performance of the system improves with the increment of the rotation angle due to mitigation of fiber nonlinearity impairments. Insets of Fig. 6(a) depict the received constellations for $\theta = 0, \pi/6$ and $\pi/4$. Fig. 6(b) illustrates BER versus OSNR for the both proposed and original OFDM system. The rotation angle is set to $\theta = \pi/4$ rad. The receiver sensitivity is obtained at a symbol rate of 25 Gsymbol/s and transmission distance of 1100 km. At $\theta = \pi/4$, the all-optical OFDM system acquires the best BER performance where the required OSNR to obtain a BER $= 1 \times 10^{-7}$ is reduced by 0.5 dB.

Finally yet importantly, we can confirm that the rotated constellation approach is a simple and viable method for reducing PAPR value, mitigating phase noise, and improving performance of OFDM systems. Furthermore, to implement this approach, only simple computations or components are required.

6. Conclusion

In this paper, an efficient new approach for mitigating the phase noise in all-optical OFDM system is proposed. This approach uses rotated constellation to reduce PAPR and mitigates the phase noise. The performance of the system is dependent on the angle difference between the odd and even subcarriers constellations. In this work, the modulation constellation of odd subcarriers is rotated clockwise with an angle of $\pi/4$, while the modulation constellation of even subcarriers is remain without rotating. By using this scheme, the PAPR is improved by 0.8 dB at CCDF $= 1 \times 10^{-3}$. Moreover, the proposed method reduces the phase noise variance due to fiber nonlinearity. In addition, at transmission distance of 1100 km and rotation angle of $\pi/4$, the required OSNR decreases by 0.5 dB for BER of $10^{-7}$.

Acknowledgment

This work was financially supported by the University of Malaya, Malaysia under various grant schemes (Grant No: D000009-16001, PG100-2014B and PG105-2014B).

References


