On Self-Clique Graphs all of whose Cliques have Equal Size

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Abstract

The clique graph of a graph $G$ is the graph whose vertex set is the set of cliques of $G$ and two vertices are adjacent if and only if the corresponding cliques have non-empty intersection. A graph is self-clique if it is isomorphic to its clique graph. In this paper, we present several results on connected self-clique graphs in which each clique has the same size $k$ for $k = 2$ and $k = 3$.

1 Introduction

Let $G$ be a graph. By a clique in $G$, we mean a maximal complete subgraph of $G$. Let $\mathcal{K}(G)$ denote the set of all cliques in $G$. The clique graph of $G$, denoted $\mathcal{K}(G)$, is the graph whose vertex set is $\mathcal{K}(G)$ and two vertices are adjacent if and only if the corresponding cliques have non-empty intersection. A graph is self-clique if it is isomorphic to its clique graph. Self-clique graphs have been the subject of much discussion lately (see [2], [3], [4], [5], [9] and [10] for instance). This paper follows in the similar vein of thought by confining the attention on those self-clique graphs whose clique sizes are uniform.

Let $\mathcal{G}(k)$ denote the set of all connected self-clique graphs where each clique is of size $k$. In the present section, we record some known results concerning $\mathcal{G}(2)$ (Theorem 1). In the next section, while unable to determine all graphs in $\mathcal{G}(3)$, we turn to determine all those in $\mathcal{G}(3)$ which are 4-regular (Corollary 3) and all those in which the degree of any vertex is

ARS COMBINATORIA 105(2012), pp. 435-449