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Constructions of Commutative Generalized Latin Squares of Order 5

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Abstract. Let $n$ be a positive integer. A generalized Latin square of order $n$ is an $n \times n$ matrix such that the elements in each row and each column are distinct. In this paper, we shall investigate classes of commutative generalized Latin squares of order 5 with 5, 13, 14 and 15 distinct elements. In addition, we shall divide the squares into equivalence classes and determine completely the squares which are embeddable in groups.

Keywords: generalized Latin squares, embeddable in groups

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INTRODUCTION

Let $n$ be a positive integer. A generalized Latin square of order $n$ is an $n \times n$ matrix such that the elements in each row and each column are distinct. Hence, a generalized Latin square of order $n$ has at least $n$ distinct elements. A generalized Latin square of order $n$ is said to be commutative if the $n \times n$ square is symmetric. Let $G$ be an additive (or multiplicative) group and let $S$ be an $n$-subset of $G$. Then the addition (or multiplication) table of $S$ will form a generalized Latin square of order $n$.

The embeddings of generalized Latin squares of order 2 and 3 in groups have been investigated by Freiman, see [1] and [2]. For the commutative case, Freiman [2] showed that there are altogether 15 squares of order 3. These squares can be divided into 7 equivalence classes and 6 of these classes are embeddable in groups. For the non-commutative case, it was shown that there are altogether 573 generalized Latin squares of order 3 and these squares can be divided into 118 equivalence classes, see [2]. Of the 118 classes, 45 classes are embeddable in groups.

The generalized Latin squares of order 4 have been studied by Tan in [3]. It was shown that there are altogether 996 commutative generalized Latin squares of order 4. These squares can be divided into 82 equivalence classes, and only 25 classes are embeddable in groups. For the non-commutative case, Tan showed that there are 20 non-commutative generalized Latin squares of order 4 with 4 distinct elements. These squares can be divided into four equivalence classes and only the squares from three equivalence classes are embeddable in groups. Tan also obtained the result that there are altogether 72 non-commutative generalized Latin squares of order 4 with 15 distinct elements. These 72 squares can be divided into 5 equivalence classes, all of which are embeddable in groups.

In this paper, we will focus on the generalized Latin squares of order 5 which are symmetric. In addition, we will generate the commutative Latin squares of order 5 with 13, 14 and 15 distinct elements as well as divide these squares into equivalence classes. There is only one commutative generalized Latin square of order 5 with 15 distinct elements:

\[
\begin{array}{ccccc}
A & B & C & D & E \\
F & G & H & I & \\
J & K & L & \\
M & N & O & \\
E & I & L & N & O
\end{array}
\]