Extension of Quickest Spectrum Sensing to Multiple Antennas and Rayleigh Channels

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***Abstract*—In this letter, we study quickest spectrum sensing for cognitive radios with multiple receive antennas in Gaussian and Rayleigh channels. We derive the probability density function for the fading case and analytically compute the upper bound and asymptotic worst-case detection delay for both of the cases. The extension into multiple antennas allows us to gain insights into the reduction in detection delay that multiple antennas can provide. Although sensing in a Rayleigh channel is more challenging, good sensing performance is still demonstrated.**

***Index Terms*—Cognitive radio, quickest spectrum sensing, CUSUM, multiple antennas, Gaussian channel, Rayleigh channel.**

I. Introduction

ONE of the most challenging tasks for cognitive radios is spectrum sensing. The most common spectrum sensing techniques are based on energy detection, matched filtering or feature detection. These techniques aim to maximize the probability of detection subject to a certain false alarm rate [1], [2]. Besides the probability of detection, detection delay is also a crucial criterion in spectrum sensing. When the primary user (PU) activity changes at some unknown point in time, it changes the distribution of the signal received by the cognitive user (CU) [3], [4]. The goal of quickest spectrum sensing [6] is to detect this abrupt change using the fewest received samples (i.e. minimize detection delay) while maintaining a certain false alarm rate.

There have been a number of studies on quickest spectrum sensing in Gaussian channels (see [3]–[5], [7], [8]) where the CUs are equipped with a single antenna. Furthermore, energy detection and generalized likelihood ratio test (GLRT) detection using multiple receive antennas has also been investigated [9]–[11]. However, no studies using quickest spectrum sensing consider CUs with multiple antennas.

In this letter, we investigate quickest spectrum sensing in two types of channels employing multiple antennas at the CU. In both cases we employ an equal gain combiner (EGC) before applying standard cumulative sum (CUSUM) sensing techniques [12]. Slowly varying channels are modeled by a time-invariant channel gain so that a Gaussian signal in noise gives an overall Gaussian received signal. Fast fading channels are modeled by a Rayleigh channel so that the received signal is the product of two complex Gaussian variables (channel and signal) with additive noise. The results provide us with new insights into the minimum detection delay that can be obtained by adding more antennas at the CU in both types of channels.

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II. System Model

In this letter, we model the signal transmitted by the PU as a narrowband complex Gaussian signal. An interweave cognitive radio network is considered where a CU monitors the channel allocated to a PU based on its own observations at each antenna. This letter focuses on the detection of the entrance of the PU to the licensed channel. The detection of the departure of a PU can be approached similarly. It is assumed that the PU is initially inactive and that the CU observes samples sequentially.

A. Combining strategy

There are various types of multi-antenna combining techniques including selection combining (SC), equal gain combining (EGC) and maximal ratio combining (MRC) [13]. In this study, it is assumed that no channel state information (CSI) is available at the receiver and therefore MRC cannot be used. Since EGC is the simplest linear diversity combining technique and performs better than SC [14] we adopt EGC as the pre-combining strategy before CUSUM detection.

B. Multi-antenna sensing with a Gaussian channel

The CU observation at antenna m is denoted by $Y_m[i]$ for $m = 1, 2, ..M$, where i is the sample number of the received signal and M is the number of antennas. If the PU is not active, $Y_m[i] = N_m[i]$, where $N_m[i]$ is independent circularly symmetric complex white Gaussian noise, $N_m[i] \sim \mathcal{CN}(0,\sigma_N^2)$. On the other hand, if the PU is transmitting, $Y_m[i] = X_m[i] + N_m[i]$, where $X_m[i] = H_m \times S_m[i]$, H_m is a time-invariant channel gain and the PU signal, $S_m[i]$, is an independent circularly symmetric complex Gaussian random variable. Hence, $X_m[i]$ is also a circularly symmetric complex Gaussian variable with variance σ_X^2. The signal-to-noise ratio (SNR) of the signal observed by the CU is σ_X^2/σ_N^2.

Figure 1 shows the spectrum sensing approach employed in this study. At each sample number, i, EGC is applied giving $z[i] = \sum_{m=1}^{M} |Y_m[i]|^2$. Then, $z[i]$ is processed by the CUSUM algorithm to determine the PU existence. Whether or not the PU signal is present, $z[i]$ has a chi-square distribution with $2M$ degrees of freedom since $Y_m[i]$ is Gaussian. Initially, the signal observed by the CU contains only noise because of the absence of the PU. At an unknown sample number, τ, the PU becomes active and the probability density function (pdf) of

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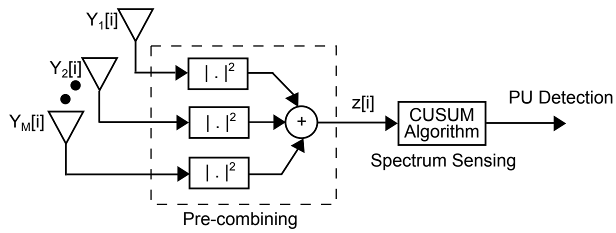

Fig. 1. Block diagram of CUSUM detection with multiple antennas employing EGC for pre-combining.

the combined signals switches instantaneously. The pdfs when the PU is absent and present are, respectively, $f_G^{(0)}(z[i]) = \frac{z[i]^{M-1}}{(\sigma_N^2)^M \Gamma(M)} e^{-\frac{z[i]}{\sigma_N^2}}$ and $f_G^{(1)}(z[i]) = \frac{z[i]^{M-1}}{(\sigma_N^2+\sigma_X^2)^M \Gamma(M)} e^{-\frac{z[i]}{\sigma_N^2+\sigma_X^2}}$, where the gamma function is $\Gamma(M) = (M-1)!$ [15].

We can then derive the log likelihood ratio $l_G(z[i]) = \ln\left\{f_G^{(1)}(z[i])/f_G^{(0)}(z[i])\right\}$ used in the CUSUM algorithm as

$$l_G(z[i]) = \frac{z[i]\sigma_X^2}{\sigma_N^2(\sigma_N^2+\sigma_X^2)} + M\ln\left\{\frac{\sigma_N^2}{\sigma_N^2+\sigma_X^2}\right\}. \quad (1)$$

The algorithm stops when a change is detected at sample $T = \inf(n : C_n \geq \gamma)$, where γ is a threshold and $C_n = \max_{k \leq n} \sum_{i=k+1}^{n} l_G(z[i])$ is the CUSUM statistic [3], [4]. The recursive form of this procedure is given by [4]

$$C_{n+1} = \{C_n + l_G(z[n+1])\}^+, n \geq 0, \quad (2)$$

where $x^+ = \max(x, 0)$ and $C_0 = 0$. Essentially, after each sample, the C_n statistic will be compared to a threshold γ and if $C_n \geq \gamma$, the algorithm will declare that the PU is present.

C. Multi-antenna sensing with a Rayleigh channel

In this scenario, if the PU is absent then $Y_m[i] = N_m[i]$. If the PU is present, $Y_m[i] = H_m[i] \times S[i] + N_m[i]$, where $H_m[i]$ is the channel coefficient, $S[i]$ is the PU signal, $N_m[i]$ is the noise, $H_m[i] \sim \mathcal{CN}(0,\sigma_H^2)$, $S[i] \sim \mathcal{CN}(0,\sigma_S^2)$ and $N_m[i] \sim \mathcal{CN}(0,\sigma_N^2)$. It is assumed that the Rayleigh fading channel is independent and identically distributed (i.i.d) between samples i and across antennas. The SNR of the received signal at the CU is $\sigma_H^2\sigma_S^2/\sigma_N^2$ and CUSUM detection is applied, as in Fig. 1.

We assume that the PU is initially inactive and at an unknown sample number, τ, it becomes active resulting in a change in the distribution of the observed signal. As in Sec. II-B, in the absence of the PU, $z[i] = \sum_{m=1}^{M} |Y_m[i]|^2$ has a chi-square distribution with $2M$ degrees of freedom and pdf given by $f_{Ray}^{(0)}(z[i]) = \frac{z[i]^{M-1}}{(\sigma_N^2)^M \Gamma(M)} e^{-\frac{z[i]}{\sigma_N^2}}$. In order to derive the pdf of the combined signal when the PU signal is present, we first derive its cumulative distribution function (cdf). Conditioned on $S[i]$, $\mathbf{Y}[i] = [Y_1[i], ..., Y_M[i]]^T$ has a complex Gaussian distribution and can be written as $\mathbf{Y}[i] = (\sigma_H^2|S[i]|^2 + \sigma_N^2)^{1/2}\mathbf{J}[i]$, where $\mathbf{J}[i]$ is an $M \times 1$ vector with independent $\mathcal{CN}(0,1)$ entries. Using this representation, $z[i] = \mathbf{Y}[i]^\dagger\mathbf{Y}[i] = (\sigma_H^2|S[i]|^2 + \sigma_N^2)\mathbf{J}[i]^\dagger\mathbf{J}[i]$. Defining $Q[i] = \mathbf{J}[i]^\dagger\mathbf{J}[i]$ and $|S[i]|^2 = \sigma_S^2 U[i]$, we have $z[i] = (\sigma_H^2\sigma_S^2|U[i]|^2 + \sigma_N^2)Q[i]$, where $U[i]$ is a standard exponential random variable and $Q[i]$ is a standard chi-square variable with $2M$ degrees of freedom. Thus, the cdf of the combined signal when the PU is present can be expressed as

$$\begin{aligned} P(z[i] < x) &= P((\sigma_H^2\sigma_S^2 U[i] + \sigma_N^2)Q[i] < x) \\ &= \mathbf{E}\left[P\left(Q[i] < \frac{x}{\sigma_H^2\sigma_S^2 U[i] + \sigma_N^2}\right)|U[i]\right] \\ &= 1 - \int_0^\infty e^{\frac{-x}{\sigma_H^2\sigma_S^2 U[i]+\sigma_N^2}} \sum_{k=0}^{M-1} \frac{1}{k!} \times \\ &\quad \left(\frac{x}{\sigma_H^2\sigma_S^2 U[i] + \sigma_N^2}\right)^k e^{-U[i]}\, dU[i]. \end{aligned} \quad (3)$$

Let $t = \sigma_T^2 U[i] + \sigma_N^2$, where $\sigma_T^2 = \sigma_H^2\sigma_S^2$, then

$$P(z[i] < x) = 1 - \frac{e^{\frac{\sigma_N^2}{\sigma_T^2}}}{\sigma_T^2} \sum_{k=0}^{M-1} \frac{x^k}{k!} \int_{\sigma_N^2}^\infty \frac{e^{\frac{-x}{t} - \frac{t}{\sigma_T^2}}}{t^k}\, dt. \quad (4)$$

The pdf of the combined signal when the PU signal is present is obtained by taking the derivative of (4) to yield

$$\begin{aligned} f_{Ray}^{(1)}(z[i]) = \frac{e^{\frac{\sigma_N^2}{\sigma_T^2}}}{\sigma_T^2} \sum_{k=0}^{M-1} \frac{z[i]^k}{k!} \int_{\sigma_N^2}^\infty \Bigg(\frac{e^{\frac{-z[i]}{t} - \frac{t}{\sigma_T^2}}}{t^{k+1}} - \\ \frac{k}{z[i]} \times \frac{e^{\frac{-z[i]}{t} - \frac{t}{\sigma_T^2}}}{t^k} \Bigg) dt. \end{aligned} \quad (5)$$

Computation of (5) is assisted by avoiding numerical integration over an infinite region. Hence, we rewrite (5) as the difference of the integral from 0 to ∞, which is given in (3.471.9) of [16, p. 363], and the finite integral from 0 to σ_N^2. Since σ_N^2 is never large, a simple Riemann sum approximation with the mid-point rule [17] works well, where rectangles are used to approximate the area under the curve. With this approach, the pdf can be written as (6), where $\mathrm{K}_\nu(.)$ is the modified Bessel function of the second kind of order ν, R is the number of rectangles and $s_r = \left(r - \frac{1}{2}\right)\left(\frac{\sigma_N^2}{R}\right)$. Numerical tests show that this approach gives a negligible error and is much faster than numerical integration with $R = 50$. The log likelihood ratio, $l_{Ray}(z[i])$ can now be calculated using the chi-square density along with (6) and the corresponding result can be substituted into (2) to detect the presence of the PU.

III. Performance Analysis

Let T denote the sample number at which the change is detected and τ be the sample number when the change actually occurs. If $T > \tau$, then the detection delay is $\delta = T - \tau$. The minimax formulation proposed by Lorden [18] models the change-point as an unknown deterministic quantity. Lorden subsequently showed that the well-known Page's CUSUM algorithm [12] is asymptotically[2] optimal in minimizing the worst-case detection delay [3], [19], [20]. Based on Lorden's formulation [18], the worst-case detection delay for Gaussian and Rayleigh channels (described in Sections II-B and II-C, respectively) are given by

$$\overline{T}_{d_G} = \sup_{\tau \geq 1} \text{ess sup } \mathbf{E}_{f_G^{(1)}}[\delta = T - \tau | T \geq \tau, z[1], ..., z[\tau]], \quad (7)$$

[2]Asymptotic here means that the mean number of samples between false alarms goes to infinity.

$$f_{Ray}^{(1)}(z[i]) = \frac{e^{\frac{\sigma_N^2}{\sigma_T^2}}}{\sigma_T^2}\sum_{k=0}^{M-1}\frac{z[i]^k}{k!}\left[2\Big(z[i]\sigma_T^2\Big)^{\frac{-k}{2}}\left(\mathrm{K}_{-k}\Big(\frac{\sqrt{z[i]}}{\frac{\sigma_T}{2}}\Big) - \frac{k\sigma_T}{\sqrt{z[i]}}\mathrm{K}_{1-k}\Big(\frac{\sqrt{z[i]}}{\frac{\sigma_T}{2}}\Big)\right) + \frac{\sigma_N^2}{R}\sum_{r=1}^{R}\frac{e^{\frac{-z[i]}{s_r}-\frac{s_r}{\sigma_T^2}}}{(s_r)^k}\Big(\frac{k}{z[i]}-\frac{1}{s_r}\Big)\right]. \quad (6)$$

$$\overline{T}_{d_{Ray}} = \sup_{\tau\geq 1}\text{ess sup }\mathbf{E}_{f_{Ray}^{(1)}}[\delta = T-\tau | T\geq\tau, z[1],...,z[\tau]], \quad (8)$$

where $\mathbf{E}_{f_G^{(1)}}$ and $\mathbf{E}_{f_{Ray}^{(1)}}$ denote the expectation operators when the change occurs at sample number τ with the pdf of $f_G^{(1)}$ and $f_{Ray}^{(1)}$, respectively[3].

Alternatively, if $T < \tau$, a false alarm event will occur with the mean number of samples to false alarm defined as $\overline{T}_{f_G} = \mathbf{E}_{f_G^{(0)}}[T]$ for the Gaussian scenario and $\overline{T}_{f_{Ray}} = \mathbf{E}_{f_{Ray}^{(0)}}[T]$ for the Rayleigh scenario [3], [4]. $\mathbf{E}_{f_G^{(0)}}$ and $\mathbf{E}_{f_{Ray}^{(0)}}$ are the expectation operators when the change never happens. The false alarm rates are then defined as $\text{FAR}_G(T) = \frac{1}{\mathbf{E}_{f_G^{(0)}}[T]}$ and $\text{FAR}_{Ray}(T) = \frac{1}{\mathbf{E}_{f_{Ray}^{(0)}}[T]}$.

The assumption of independence between the received signals allows us to express the lower bound on the mean number of samples between false alarms as $\overline{T}_{f_G} \geq e^{\gamma}$, and $\overline{T}_{f_{Ray}} \geq e^{\gamma}$ [4], [18], [20]. We now proceed to derive the upper bound for the worst case detection delay for the Gaussian channel, $\overline{T}_{d_G}$. Using Wald's equation, Theorem 1 in [21] and the fact that the combined signal, $z[i]$ follows a chi-square distribution, we can express the upper bound of $\overline{T}_{d_G}$ as

$$\overline{T}_{d_G} \leq \frac{\gamma+\varphi}{D(f_G^{(1)}||f_G^{(0)})}, \quad (9)$$

where

$$\varphi = \frac{\int_{\upsilon}^{\infty} l_G^2(z) f_G^{(1)}(z)\,dz}{D(f_G^{(1)}||f_G^{(0)})}, \quad (10)$$

$$\upsilon = -\frac{\sigma_N^2(\sigma_N^2+\sigma_X^2)}{\sigma_X^2}\times M\ln\left\{\frac{\sigma_N^2}{\sigma_N^2+\sigma_X^2}\right\}. \quad (11)$$

Note that υ is the zero of the log likelihood ratio function, $l_G(z[i])$ and $D(f_G^{(1)}||f_G^{(0)})$ is the Kullback-Leibler divergence of $f_G^{(1)}$ from $f_G^{(0)}$ which can be defined as

$$\begin{aligned} D(f_G^{(1)}||f_G^{(0)}) &= \int_0^{\infty} f_G^{(1)}(z)\ln\left\{\frac{f_G^{(1)}(z)}{f_G^{(0)}(z)}\right\}dz \\ &= -M\ln\{\sigma_N^2+\sigma_X^2\} + \frac{M\sigma_X^2}{\sigma_N^2} + M\ln\{\sigma_N^2\}. \end{aligned} \quad (12)$$

The upper bound for the worst case detection delay for the Rayleigh channel, $\overline{T}_{d_{Ray}}$ can be written with the aid of Wald's equation and Theorem 1 in [21] along with the fact that the combined signal from each antenna, $z[i]$ is chi-square distributed, which yields

$$\overline{T}_{d_{Ray}} \leq \frac{\gamma+\Phi}{D(f_{Ray}^{(1)}||f_{Ray}^{(0)})}, \quad (13)$$

in which

$$\Phi = \frac{\int_{\eta}^{\infty} l_{Ray}^2(z) f_{Ray}^{(1)}(z)\,dz}{D(f_{Ray}^{(1)}||f_{Ray}^{(0)})}, \quad (14)$$

where η, the zero of $l_{Ray}(z[i])$, and $D(f_{Ray}^{(1)}||f_{Ray}^{(0)})$, the Kullback-Leibler divergence, are calculated numerically.

As $\overline{T}_{f_G}, \overline{T}_{f_{Ray}} \to \infty$, $\gamma \to \infty$. Therefore, it is desirable in practice to analyze the detection performance asymptotically as it will give a low false alarm rate [20]. Asymptotically, the worst case detection delay can be approximated using Theorem 1 in [18]. Hence, the asymptotic detection delay, for Gaussian and Rayleigh channels are given by

$$\overline{T}_{d_G} \sim \frac{\gamma}{D(f_G^{(1)}||f_G^{(0)})}, \quad \overline{T}_{d_{Ray}} \sim \frac{\gamma}{D(f_{Ray}^{(1)}||f_{Ray}^{(0)})}. \quad (15)$$

IV. Analytical Results and Simulation

In this section, we present some analytical and simulated results to evaluate the sensing performance in Gaussian and Rayleigh channels employing multiple receive antennas at the CU. We consider the case when the PU starts transmitting at $\tau = 100$. Simulations use 50000 trials to generate each point, where each trial has 200 samples. The thresholds for both cases are set using the respective lower bounds on the mean number of samples between false alarms, $\overline{T}_{f_G}$ and $\overline{T}_{f_{Ray}}$. The number of rectangles required for the Riemann sum in (6) is set to R = 50. Figure 2 compares the simulated and analytical results for the case when the CU is equipped with single and multiple antennas (M = 3) and the Gaussian signal is observed over a time-invariant channel at SNR = 4.77dB[4]. We observe that with M = 3 antennas at the CU, the detection delay, $\overline{T}_{d_G}$ reduces substantially compared to M = 1. The reduction in detection delay is at least 6 samples which represents at least a 6-fold improvement.

In Figure 3, we present a comparison between the sensing performance in Gaussian and Rayleigh channels at SNR = 10dB when the CU is equipped with M = 3 antennas. Comparing $\overline{T}_{d_G}$ with $\overline{T}_{d_{Ray}}$ in Figure 3, we can see that the sensing performance degrades in the Rayleigh fading channel as compared to a Gaussian channel due to the faded received signals. However, the impact of the propagation conditions is small compared to the effects of different numbers of antennas as shown from a comparison of Figures 2 and 3.

Figure 4 shows the performance improvements due to adding more antennas at the CU for the Gaussian and the Rayleigh scenarios. The improvements are due to the spatial diversity provided.

In all of the cases shown in Figures 2 and 4, the simulation results are close to the asymptotic analysis and the upper bound is very loose. Hence, the asymptotic result provides a good indicator of sensing performance. Deviations of the

[3]Essential supremum (ess sup) is used in (7) and (8) so that $\overline{T}_{d_G}$ and $\overline{T}_{d_{Ray}}$ takes the worst-case value over all possible realizations of the z's before the change [19].

[4]The SNR value of 4.77dB is chosen for comparison purposes with [4].

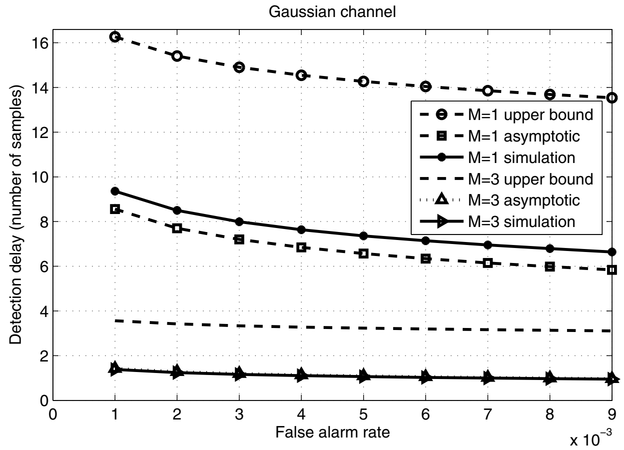

Fig. 2. Comparison between the performance of the CU equipped with single (M = 1) and multiple (M = 3) antennas for a Gaussian channel with SNR = 4.77dB.

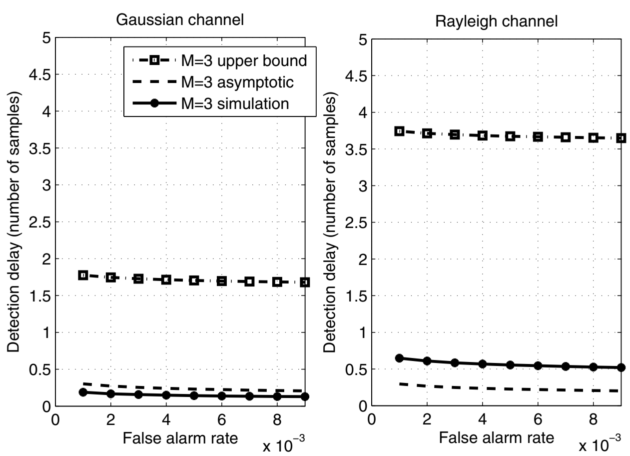

Fig. 3. Simulation and analytical sensing performance in Gaussian and Rayleigh channels at SNR = 10dB.

asymptotic results from the simulations are due to the fact that the theory involves a large threshold value and an asymptotically small false alarm rate.

V. Conclusion

In this letter, we have studied quickest spectrum sensing for CU with multiple antennas when the received signals experience Gaussian and Rayleigh fading channels. We derived the pdf for the Rayleigh fading scenario. In addition, we also derived an analytical performance analysis for both scenarios and evaluated the performance of the spectrum sensing in terms of detection delay and false alarm rate. The numerical analysis and simulation results illustrate the performance gain that can be achieved in quickest spectrum sensing in Gaussian and Rayleigh channels by having more antennas at the CU. We showed the effect of propagation conditions on the sensing performance. Although the detection delay is slightly higher in the Rayleigh fading channel compared to the Gaussian channel, we still observed a fairly good sensing performance.

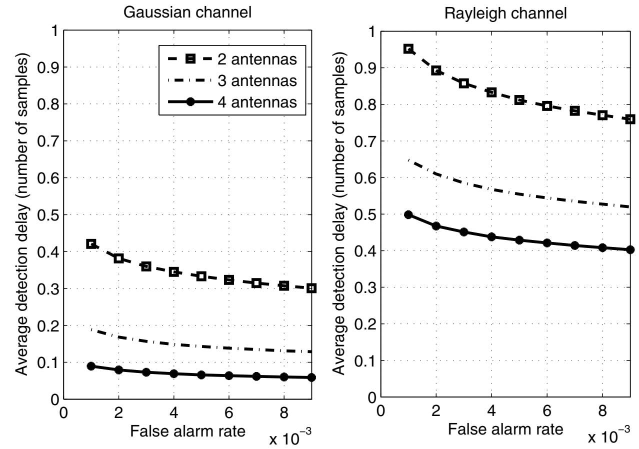

Fig. 4. The effect of different number of antennas on the simulated performance of sensing in Gaussian and Rayleigh channels at SNR = 10dB.

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