A New Technique to Derive Invariant Features for Unequally Scaled Images

P.Raveendran[+], Sigeru Omatu* and Poh Sin Chew[+]

[+]Engineering Faculty
Universiti Malaya
Kuala Lumpur, 59100
Malaysia.

*Dept. of Computer and System Science
College of Engineering
University of Osaka Perfecture,
Sakai, Osaka, 593, Japan.

ABSTRACT

This paper presents a new technique to derive features for images that are translated, scaled equally/unequally and rotated. The problem is formulated using conventional regular moments. It is shown that the conventional regular moment-invariants remain no longer invariant when the image is scaled unequally in the x- and y-directions. A method is proposed to form moment-invariants that do not change under such unequal scaling. The newly formed moments are also invariant to translation, and reflection. However, it is not invariant for images that are rotated. A neural network is trained to estimte the angle of rotation and by using it invariant moments for images that are unequally scaled, translated and rotated are derived.. Computer simulation results are also included to show the validity of the method proposed.

1. INTRODUCTION

The automatic recognition of an object in a scene regardless of its position, size and orientation is an important problem in pattern analysis. A number of techniques have been developed to derive features from an image which are invariant under translation, scale change and rotation [1-2]. In particular, the invariant properties of regular moments have attracted many users to utilise them as pattern features in object recognition, pattern classification and scene matching [3-5]. Hu [4] published his classic paper on pattern recognition by deriving a set of regular moment invariants based on combinations of regular moments using algebraic invariants. Besides Hu, Bamieh and De Figueiredo [9] derived another set of moment invariants using the theory of algebraic invariants. The main characteristics of the invariants formulated by Bamieh et.al is that the feature vector size is much lower than any other known invariants, which makes them computationally cheaper. These regular moments are invariant to changes in scale, shift, rotation and reflection.

The authors in [4] and [9] have shown that the moments remain invariant when the scale change in the x and y-directions is equal and the derived features have been used for pattern classification of ship [5] and other applications [6-8]. In some of the applications the scale change in the x- and y- directions is not equal. This maybe due to the digital nature of the imagery caused by undersampling and digitizing effects. Or possibly the image itself in comparison with the standard image has unequal scale change in the x- and y- directions. So, when the image is an elongated or compressed version of the original image, as illustrated in Figs. (1a) and (1b), then these moments do not remain invariant.

We have formed moment invariants that are invariant to unequal scaling in the x- and y-directions based on the regular moments [8]. However, they are only invariant to scale, translation, and reflection. If the scaling constants for the x and y-directions are equal then rotation invariance can be achieved by combining the moments based on the theory of algebraic invariance as shown in [4]. Rotation invariance can also be achieved by knowing the angle of rotation and using the relationship between the rotated image and its original form. The angle is computed by using the principal axis method [7]. This is done by using the second order moments. However, this method cannot be used for images that are unequally scaled because the angle of rotation is dependent on the equal scale change in the x- and y-directions.

To overcome this problem we used a three layered neural network trained to estimate the angle of rotation. The back-propagation learning algorithm is used in training the neural network [10]. The neural net is presented with second order moments of some of the unequally scaled images and are tested with the remaining images. The scaling constants for the x and y-directions are between 0.8 and 1.2 and the images are rotated through 0°, 30°, 60°, 150°, 180°, 225° and 300°.

2. BASIC THEORY AND NEW REGULAR MOMENTS

Figure 1(a) shows an original image while Fig. 1(b) shows after it is shifted and unequally scaled in the x and y directions. The conventional regular moments can be defined as :

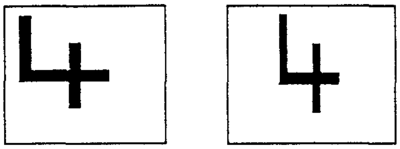
$$m_{pq} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^p y^q f(x,y)dxdy, \quad \text{for } p,q = 0,1,2,\dots\dots\dots \qquad (1)$$

0-7803-4053-1/97/$10.00 © 1997 IEEE

In this paper we limit ourselves to binary images. For binary images f(x,y) is either 1 or 0. Assume the region of interest to be evaluated is defined as follows :

$$m_{pq} = \int_{y_1}^{y_2} \int_{x_1}^{x_2} x^p y^q f(x,y) dx dy$$

for p,q = 0,1,2,..... (2)



(a) (b)

Fig. 1(a):Original image f(x,y). and (b) Unequally scaled and shifted.image of (a)

To make these moments invariants to translation, one can define the central moments as

$$\mu_{pq} = \int_{y_1}^{y_2} \int_{x_1}^{x_2} (x - \bar{x})^p (y - \bar{y})^q f(x,y) dx dy \qquad (3)$$

where $\bar{x}$ and $\bar{y}$ are the coordinates of the centroid given by

$$\bar{x} = \frac{m_{10}}{m_{00}}, \qquad \bar{y} = \frac{m_{01}}{m_{00}} \qquad (4)$$

Solving for $\bar{x}$ and $\bar{y}$ and substituting them into (3) gives

$$\mu_{pq} = \int_{y_1}^{y_2} \int_{x_1}^{x_2} \left(x - \frac{1}{2}(x_1 + x_2)\right)^p \left(y - \frac{1}{2}(y_1 + y_2)\right)^q f(x,y) dx dy \qquad (5)$$

Using the binomial expansion, (5) can be expressed as

$$\mu_{pq} = \left[\frac{(x_2 - x_1)^{p+1}}{p+1} + \ldots + C_p^p (x_2 - x_1)\left(-\frac{1}{2}(x_1 + x_2)\right)^p\right] \times \left[\frac{(y_2 - y_1)^{q+1}}{q+1} + \ldots + C_q^q (y_2 - y_1)\left(-\frac{1}{2}(y_1 + y_2)\right)^q\right] \qquad (6)$$

These moments are made invariant to scale change as proposed in [3] by

$$\eta_{pq} = \frac{\mu_{pq}}{\mu_{00}^{\frac{(p+q)}{2}}} \qquad (7)$$

Now, consider the unequally scaled image in Fig. 1(b). Assume the expansion in the x and y direction to be α and β respectively. The central moments can be defined as

$$\tilde{\mu}_{pq} = \left[\frac{\alpha^{p+1}(x_2 - x_1)^{p+1}}{p+1} + \ldots + C_p^p (x_2 - x_1)\alpha \frac{1}{2}(\alpha\chi_1^2 + \alpha\chi_2^1\ ))^p\right] \times \left[\beta^{q+1}\frac{(y_2 - y_1)^{q+1}}{q+1} + \ldots + C_q^q (y_2 - y_1)\beta \frac{1}{2}(\beta y_2 + \beta y_1))^q\right] \qquad (8)$$

Evaluating further (8), the central moments can be expressed in terms of the original as

$$\tilde{\mu}_{pq} = \cdot\alpha^{p+1} \beta^{q+1} \mu_{pq} \qquad (9)$$

If now $\tilde{\eta}_{pq}$ is evaluated for the unequally scaled image in Fig. 1(b) and expressing in terms of the original image, we get

$$\tilde{\eta}_{pq} = \left(\frac{\beta}{\alpha}\right)^{\frac{q-p}{2}} \eta_{pq} \qquad (10)$$

In the above equation the sign '~' denotes the moments formed for the unequally scaled image. It is evident from (10) that when $\alpha = \beta$, $\tilde{\eta}_{pq}$ is same as η_{pq}, thus giving us moments which are invariant. The authors in [4-7] have all used $\alpha = \beta$ in solving their problems. Earlier, we have proposed a technique to make it invariant when $\alpha \neq \beta$ [8]. However, in [8], p and q must be greater or equal to 2. In this paper, there are no restrictions on the values of p and q can take. In order to form moment-invariants when $\alpha \neq \beta$, we cosider the following

$$\gamma_{pq} = \frac{\eta_{pq}}{\eta_{p+1,q+1}} \qquad \text{for p,q} = 0,1,2,3,\ldots.. \qquad (11)$$

and

$$\tilde{\gamma}_{pq} = \frac{\tilde{\eta}_{pq}}{\tilde{\eta}_{p+1,q+1}} \qquad \text{for p,q} = 0,1,2,3,\ldots.. \qquad (12)$$

where once again '~' refers to moments evaluated for unequally scaled image as shown in Fig. 1(b). Substituting (10) into (12), it is evident that $\tilde{\gamma}_{pq} = \gamma_{pq}$, for p,q = 0,1,2,... giving us moment invariance even when $\alpha \neq \beta$.

3. COMPUTER SIMULATIONS

The images shown in Table 1 were drawn onto a 256x256 grid. The image '4 ' depicted in Fig. 1(a) was considered and the moments for this were evaluated and tabulated in row 2 of Table 1. Similarly, the moments for the unequally scaled and shifted image shown in Fig. 1(b) for various values of α and β were evaluated. These results are again tabulated in rows 3 to 7 of Table 1. The close agreement obtained in each of these columns verify the correctness of the proposed technique. The reason for not obtaining exact invariances for each column is that the image function is digital rather than continous.

Since in the foregoing analysis, no assumption was made regarding the values that α or β may assume, we may allow them to become negative. This evidently treats the appropriate mirror reflections. The results are tabulated in Table 2. Once again, the constancy of values obtained in each of the columns demonstrate the validity of the proposed schemes.

Table 1: Scaled invariant features for equally and unequally scaled images. α and β correspond to scale change in the x- and y- directions respectively.

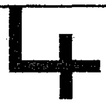
Scale	image	γ_{20}	γ_{02}	γ_{11}	γ_{21}	γ_{12}	γ_{30}	γ_{03}
α=1.0 β=1.0		3.5777	2.2286	1.7794	0.4785	1.1780	2.4226	1.0734
α=1.0 β=0.8		3.5576	2.2220	1.7750	0.4710	1.1742	2.3693	1.0697
α=1.2 β=0.8		3.5957	2.2455	1.7924	0.4733	1.1854	2.3649	1.0807
α=0.8 β=1.2		3.5354	2.1974	1.7562	0.4796	1.1638	2.4704	1.0594
α=1.2 β=1.0		3.6156	2.2523	1.7968	0.4809	1.1892	2.4171	1.0844
α=1.2 β=1.2		3.6289	2.2568	1.7998	0.4859	1.1917	2.4529	1.0869

Table 2: Moments of the images obtained by reflection. Rows 1,2,3, and 4 refer to the moments for the original image, and image reflected about x-axis, about y-axis and both axes respectively.

images	reflection	γ_{20}	γ_{02}	γ_{11}	γ_{21}	γ_{12}	γ_{30}	γ_{03}
	original	9.7141	8.5638	1.2819	0.4617	6.6871	7.6125	2.1030
	x-axis	9.7141	8.5638	1.2819	0.4617	6.6871	7.6125	2.1030
	y-axis	9.7141	8.5638	1.2819	0.4617	6.6871	7.6125	2.1030
	both axes	9.7141	8.5638	1.2819	0.4617	6.6871	7.6125	2.1030

4. ROTATION PROPERTY AND NEURAL NETWORK

The new regular moments, γ_{pq} are invariant under change of size, translation and reflection. They are not invariant to rotation and it is easily seen why it isn't when expressed in the polar form. We assume the coordinate origin has been chosen to coincide with the image centroid. The moments, m_{pq}, in the polar form is

$$m_{pq} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(r\cos\theta)^p (r\sin\theta)^q rf(r,\theta)drd\theta \quad \text{for } p,q = 0,1,2,\ldots\ldots\ldots \tag{13}$$

where $x = r\cos\theta$ and $y = r\sin\theta$ and the Jacobian of the transformation is r.

Now, consider the image has rotated through an angle ϕ. If the rotated image is denoted by f^r, the relationship between the original and the rotated image in the same polar coordinates is

$$f^r(r, \theta) = f(r, \theta - \phi) \tag{14}$$

By a change of variable $\varphi = \theta - \phi$, the moments for the rotated image, m^r_{pq}, is

$$m^r_{pq} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(r\cos(\phi+\varphi))^p (r\sin(\phi+\varphi))^q rf(r,\varphi)drd\varphi \tag{15}$$

As can be seen in Eq. (15), the angle of rotation is used in computing the moments when an image is rotated. By combining the moments based on the theory of algebraic invariants rotation invariance can be realized [4]. However, this technique is only applicable when the scale changes in the x- and y- directions are equal.

A relationship between the original image, m_{pq}, and the rotated image m^r_{pq}, can be established by expanding Eq. (15) and expressing it in terms of the moments of the original image. If the image rotates through an angle ϕ, the moments change according to

$$m'_{pq} = \sum_{r=0}^{p}\sum_{s=0}^{q}(-1)^{q-s}\binom{p}{r}\binom{q}{s}(\cos\phi)^{p-r+s}(\sin\phi)^{q+r-s}(m_{p+q-r-s,r+s}) \tag{16}$$

The angle of rotation, ϕ, is determined by using the principle of axis method which uses the second order moments [7]. The angle of rotation, ϕ, is computed from

$$\phi = \frac{1}{2}\tan^{-1}\frac{2m_{11}}{m_{20} - m_{02}} \tag{17}$$

However, the angle of rotation is significant only when the scale changes in the x and y-directions are equal. Figure 2 shows an example of a class of image numeral 2 that uses different scaling constants. The angle of rotation for each image "at rest" shown in Figure 2 is different though the images belong to the same class. Figure 3 shows an example of an image in Figure 2 that has been rotated.

A neural network is trained to generate the angle of rotation for each one of the images shown in Figure 2. The neural network shown in Figure 4 uses only the second order moments as inputs and the target value of each image are determined by Eq. (17). The neural network is tested with images as shown in Figure 5 have scaling constants between 0.9 and 1.2. The results are tabulated in Table 3.

By knowing how much an image has rotated from its original form and by using Eq. (18), only the unequal/equal scaling constants in the x- and y-directions are left to be eliminated. This is eliminated by using Eq. (11).

$$m^{ur}_{pq} = \sum_{r=0}^{p}\sum_{s=0}^{q}(-1)^{p+q-r}\binom{p}{r}\binom{q}{s}(\cos\phi)^{p-r+s}(\sin\phi)^{q+r-s}(\gamma_{p+q-r-s,r+s}) \tag{18}$$

where m^{ur}_{pq} is the new regular moment that is invariant to rotation. Table 3 shows the results of the images shown in Figure 5 and six rotated versions with rotation angles of 30°, 60°, 150°, 180°, 225° and 300°. The mean value for each angle of rotation is also tabulated. The sample mean, μ, sample standard deviation, σ, and $\sigma/\mu\%$, which indicates the percentage spread of the λ_{pq} values from the corresponding means are also indicated. The reason for not obtaining the exact invariances is the discrete form of the image function rather than being a continuous one.

5. Conclusion

The regular moments, besides invariant to translation, rotation and reflection are only invariant to scale if the scale changes in the x- and y-directions are equal. In this paper, we have proposed a new method the new regular moments are invariant to unequally/equally scaled, translation and reflection. However, they are not invariant to rotation. To solve this problem a three layered neural network is trained with second order moments to generate the angle of rotation. By using the angle obtained from the neural network and using Eq. (18) a set of moment invariants for second and third order is derived for images that are unequally scaled, translated, and rotated. The results are shown in Table 3. Though exact values are not obtained the technique of using a neural network to generate the angle offers a solution to solving images that are unequally scaled and rotated. Preliminary investigation about the performance of this method in the presence of noise shows the method is promising.

Table 3: Magnitudes of moment invariant for the images shown in Fig. 5 and rotated by the angles shown in the Table. The angle of rotation is denoted by. ϕ

scale	λ_{20}	λ_{02}	λ_{11}	λ_{21}	λ_{12}	λ_{30}	λ_{03}
φ=0°	3.5777	2.2286	1.7794	0.4785	1.1780	2.4226	1.0734
φ=30°	3.5576	2.2220	1.7750	0.4710	1.1742	2.3693	1.0697
φ=60°	3.5957	2.2455	1.7924	0.4733	1.1854	2.3649	1.0807
φ=150°	3.5354	2.1974	1.7562	0.4796	1.1638	2.4704	1.0594
φ=180°	3.5766	2.2223	1.7968	0.4809	1.1792	2.4171	1.0844
φ=225°	3.3221	2.2601	1.7945	0.4655	1.1745	2.4242	1.0763
φ=300°	3.6289	2.2568	1.7998	0.4859	1.1917	2.4529	1.0869
μ	3.5420	2.2332	1.7849	0.4764	1.1781	2.4173	1.0758
σ	0.1013	0.0223	0.0156	0.0069	0.0089	0.0392	0.0094
σ/μ%	0.0285	0.0099	0.0087	0.0144	0.0075	0.0162	0.0087

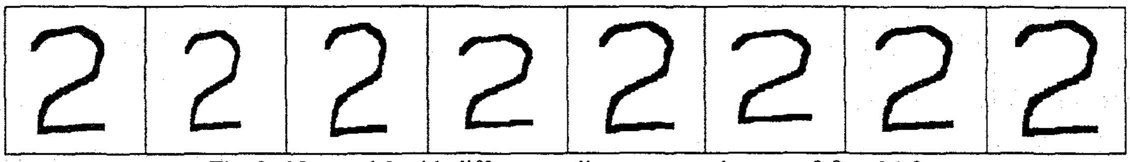


Fig. 2. Numeral 2 with different scaling constants between 0.8 and 1.2

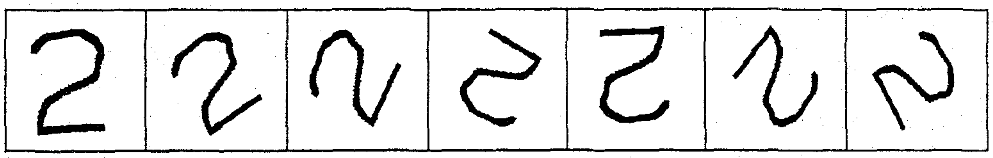


Fig.3: Numeral 2 rotated through 0°, 30°, 60°, 150°, 180°, 225° and 300°.

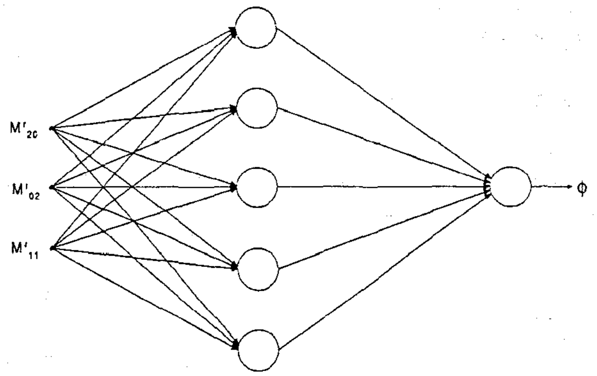


Fig. 4. Neural network model to estimate the angle ϕ

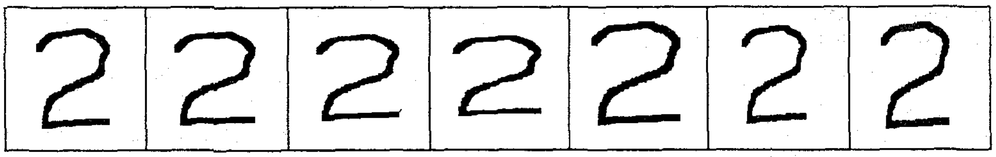


Fig 5: Numeral 2 with varying scaling constants between 0.9 and 1.2

REFERENCES

[1] E. Persoon and K.S. Fu, "*Shape discrimination using Fourier descriptors*", IEEE. Trans. Systems, Man and Cybernatics, Vol. SMC-7, pp. 170-179, 1977.

[2] Y.N. Hsu, H.H. Arsenault, and G. April, "*Rotational invariant digital pattern recognition using circular harmonic expansion*", Applied Optics, Vol. 21, pp. 4012-4015, 1982.

[3] R.O. Duda and P.E. Hart, "Pattern classification and scene analysis", New York: Wiley, 1973.

[4] M.K. Hu, "*Visual pattern recognition by moment invariants*", IRE Trans.Information Theory, Vol. IT-8, pp. 179-187, February 1962.

[5] F.W. Smith and M.H. Wright "*Automatic ship photo interpretation by the method of moments*", IEEE Trans. Computer, C-20, pp. 1089-1094, 1971.

[6] S.A. Dudani, K.J. Kenneth and R.B. McGhee, "*Aircraft identification by moment invariants*", IEEE Trans. Computer, C-26, pp. 39-45, 1977.

[7] M.R. Teague, "*Image analysis via general theory of moments*", Journal of Optical Society of America, Vol. 70, pp. 920-930, August 1980.

[8] P.Raveendran, and S. Omatu, "*Neuro-pattern classification of elongated and contracted images*", Information Sciences: Vol. 3 pp. 209-221, 1995.

[9] R.Bamieh and De. Figueiredo, "A general moments/ attributed-graph method for the three dimensional object recognition from a single image, IEEE Journal of Robotics and Automation Vol. 2, pp. 31-41, 1986.

[10] D.E. Rumelhart and J.L. McCelland, Parallel Distributed Processing: Exploration in the Microstructure of Cognition, MIT Press, Cambridge, MA, Vol 1, 1986.