Evolution of an Airy beam in a saturated medium

2014 Laser Phys. 24 115402

(http://iopscience.iop.org/1555-6611/24/11/115402)

IOP Publishing | Astro Ltd Laser Physics

Laser Phys. **24** (2014) 115402 (6pp) doi:10.1088/1054-660X/24/11/115402

Evolution of an Airy beam in a saturated medium

Rui-Pin Chen[1], **Khian-Hooi Chew** [2], **Ting-Yu Zhao**[1], **Pei-Gang Li**[1], **Chao-Rong Li**[1]

[1] Department of Physics, Zhejiang Sci-Tech University, Hangzhou, 310018, People's Republic of China
[2] Department of Physics, Faculty of Science, University of Malaya, Kuala Lumpur 50603, Malaysia

E-mail: chenrp@zstu.edu.cn

Received 1 July 2014, revised 20 July 2014
Accepted for publication 27 July 2014
Published 3 September 2014

Abstract

The nonlinear dynamics of an Airy beam in a saturated medium is presented. An analytical expression for the evolution of the Airy beam width in the root-mean-square sense is derived. The novel features of the collapsing beams of an Airy beam in a saturated medium are demonstrated by numerical calculation. These collapsing beams shift laterally and are the main property of the Airy beam. However, the collapsing beam in the major lobe of an Airy beam tends to shift in the opposite direction for conservation of the beam centroid. The location and evolution of the collapsing beams depend strongly on the initial powers. The peak intensities of the collapsing beams oscillate at almost the same intensity in the saturated medium, regardless of their initial powers. These results are useful for manipulating nonlinear wave collapse and multi-filamentation.

Keywords: saturated medium, Airy beam, collapsing beam

(Some figures may appear in colour only in the online journal)

1. Introduction

Nonlinear wave collapse has been investigated in many areas of physics, including optics, fluidics, plasma physics and Bose–Einstein condensates [1, 2]. In particular, the collapse dynamics of an optical field has attracted significant attention due to the complex and universal phenomena associated with the process [3]. Collapse dynamics and filamentation of some novel optical beams such as vortices, Airy beams and ultra-short laser pulses have been investigated both theoretically and experimentally in numerous contexts [4–7]. The beams undergo collapse in a transparent self-focusing medium when the input power surpasses a certain critical power. If the initial power is very high, other nonlinear effects that give rise to modulation instability such as higher-order nonlinearity, self-steepening and self-phase modulation should be considered. Higher-order processes such as plasma generation halt beam collapse, which breaks the beam up into multiple filaments [1–3]. The spatial dynamics of multiple filaments and their interaction have been demonstrated [8]; however, it still remains a challenging task to effectively control and manipulate optical field collapse and multi-filamentation.

The Airy beam has attracted immense interest since the first experimental generation of Airy beams by Siviloglou *et al* [9] due to its unique features such as diffraction-free and transverse acceleration [10], self-healing [11] and potential applications [12–15]. Most work has been performed under paraxial conditions. Recently, nonparaxial Mathieu and Weber accelerating linear case beams have been demonstrated. As generalized nonparaxial accelerating beams, nonparaxial Mathieu and Weber accelerating beams certainly provide greater possibility for launching and controlling the desired beam trajectories [16]. In our previous works [5, 17], the occurrence and control of the collapse of an Airy beam and a vortex Airy beam in a focusing Kerr medium were investigated. The use of a nonlinear medium with strong saturation and no collapse (filamentation) was recently considered as well [18]. To the best of our knowledge, the nonlinear dynamics of an Airy beam in a saturated medium [19–23] have not been studied in detail. In this work, we study the nonlinear dynamic behaviours of a coherent Airy beam in a medium with a 'soft' nonlinearity saturation as demonstrated in [23]. As the initial power increases, energy tends to accumulate in the beam centre due to the effect of self-focusing nonlinearity. Higher initial power leads to more collapsing beams appearing at the outermost lobes. Furthermore, these collapsing beams shift laterally as a result of the spatial profile of the Airy beam. On the other hand, the collapsing beam in the major lobe of an Airy beam tends to shift in the opposite direction for conservation of the beam centroid. The peak intensities of the collapsing beams oscillate at almost the same intensity in the saturated medium, regardless of their initial powers. These results provide an effective approach for tailoring the collapsing beams and multi-filamentation in an Airy beam.

2. Theoretical formulation

The nonlinear dynamic behaviour of an Airy beam in a saturated medium can be described by a two-dimensional (2D) nonlinear Schrodinger (NLS) equation with a focusing cubic nonlinearity and a fifth-order weak defocusing nonlinearity as follows

$$\nabla_{\perp}^{2} E - 2ik\frac{\partial E}{\partial z} + \frac{2n_2k^2}{n_0}|E|^2 E - \frac{2n_4k^2}{n_0}|E|^4 E = 0 \quad (1)$$

where $\nabla_{\perp}^{2} = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the transverse Laplacian, k is the linear wave number, z is the longitudinal coordinate, n_0 is the linear refraction index of the medium, n_2 is the third-order nonlinear coefficient and n_4 is the fifth-order nonlinear coefficient. Note here that fifth-order Kerr nonlinearity has been added in the NLS equation to capture the beam collapse and allow propagation of the beam through the collapsing point [8, 23].

Two important invariants of the NLS equation are the beam power

$$P(z) = \iint_s |E|^2 \,\mathrm{d}x\mathrm{d}y \quad (2)$$

and the Hamiltonian

$$H(z) = \frac{1}{4k^2}\iint_s \left[|\nabla E|^2 - \frac{k^2n_2}{n_0}|E|^4 + \frac{2k^2n_4}{3n_0}|E|^6\right]\mathrm{d}x\mathrm{d}y\,. \quad (3)$$

An additional invariant required by the NLS during evolution is the centroid.

$$I_{0,0}(z) = \iint_s (x+y)|E|^2 \,\mathrm{d}x\mathrm{d}y\,. \quad (4)$$

The beam width in the root-mean-square (rms) sense can be defined as

$$W(z) = \iint_s (x^2+y^2)|E|^2 \,\mathrm{d}x\mathrm{d}y, \quad (5)$$

and solutions of the NLS equation satisfy the following relation [5, 24]:

$$W(z) = W(z=0) + \frac{dW(z=0)}{dz}z + 4H(z=0)z^2\,. \quad (6)$$

By taking an Airy beam as an initial field distribution [9], we have

$$E(x,y;z=0) = A_0 Ai(x/x_0)\,Ai(y/y_0)\,\exp\left(a_x x/x_0 + a_y y/y_0\right) \quad (7)$$

where A_0 is the amplitude of the complex amplitude $E(x, y, z = 0)$ and x_0 is an arbitrary transverse scale. Exponential factors a_x and a_y are set as positive volumes to ensure the finite energy of the infinite Airy tail in the x- and y-directions, respectively. When $H(z) = 0$ (see equation (6)), the rms beam width becomes constant, which indicates competition among the third-order self-focusing effect, the fifth-order defocusing effect and the diffraction effect. If the higher order nonlinear effect is neglected (i.e. $n_4 = 0$), the critical power of the Airy beam in a Kerr medium required for the optical field to collapse, P_{cr}, can be obtained from equations (2)–(7) as [17]

$$P_{cr} = \frac{a_x + a_y}{16a_x^2a_y^2K_0(2a_x^3)\,K_0\left(2a_y^3\right)\exp\left(2a_x^3 + 2a_y^3\right)}P_{cr}^{G} \quad (8)$$

where $P_{cr}^{G} = 2\pi n_0/(n_2k^2)$ is the critical power of a Gaussian beam. Equation (8) clearly indicates that the critical power of a vortex vector beam depends only on the profile of the Airy beam, which is related to the beam parameters a_x and a_y. In this case, competition exists between the self-focusing Kerr effect and the defocusing effect of diffraction to determine the critical power of the Airy beam. When the initial power equals the critical power, the beam rms width remains invariant. This implies total balance between the self-focusing Kerr effect and the defocusing effect of diffraction. As initial power exceeds critical power, the beam rms width decreases and goes to zero at a finite propagation distance, followed by a global collapse. Nevertheless, the evolution of the beam in each position and the occurrence of partial collapse during propagation cannot be determined accurately, especially for an irregular beam such as the Airy beam. In general, partial collapse occurs in a certain position before the total rms beam width approaches a constant value. Partial collapse may also occur before the total rms beam decreases and eventually goes to zero as initial power reaches and exceeds critical power [25, 26]. It is rather complicated to predict the appearance and evolution of collapsing beams in a saturated medium due to the co-existence of the third-order self-focusing effect, the fifth-order defocusing effect and the diffraction effect. The appearance and evolution of the collapsing beams in each position during propagation in the saturated medium can be studied only numerically, which is discussed in the following section.

3. Numerical results and analysis

In this work, the 2D NLS equation is numerically solved by a split-step finite-difference method [27–29] that combines the split step beam propagation method with the finite-difference method for the nonlinear and diffractive terms. Let us take $\lambda = 0.53\,\mu$m and $x_0 = y_0 = 10\,\mu$m and $z_0 = kx_0^2/2 = 0.6$ mm and $n_4 = 0.05n_2$. The intensity distributions of the Airy beam in the saturated medium with various initial powers at

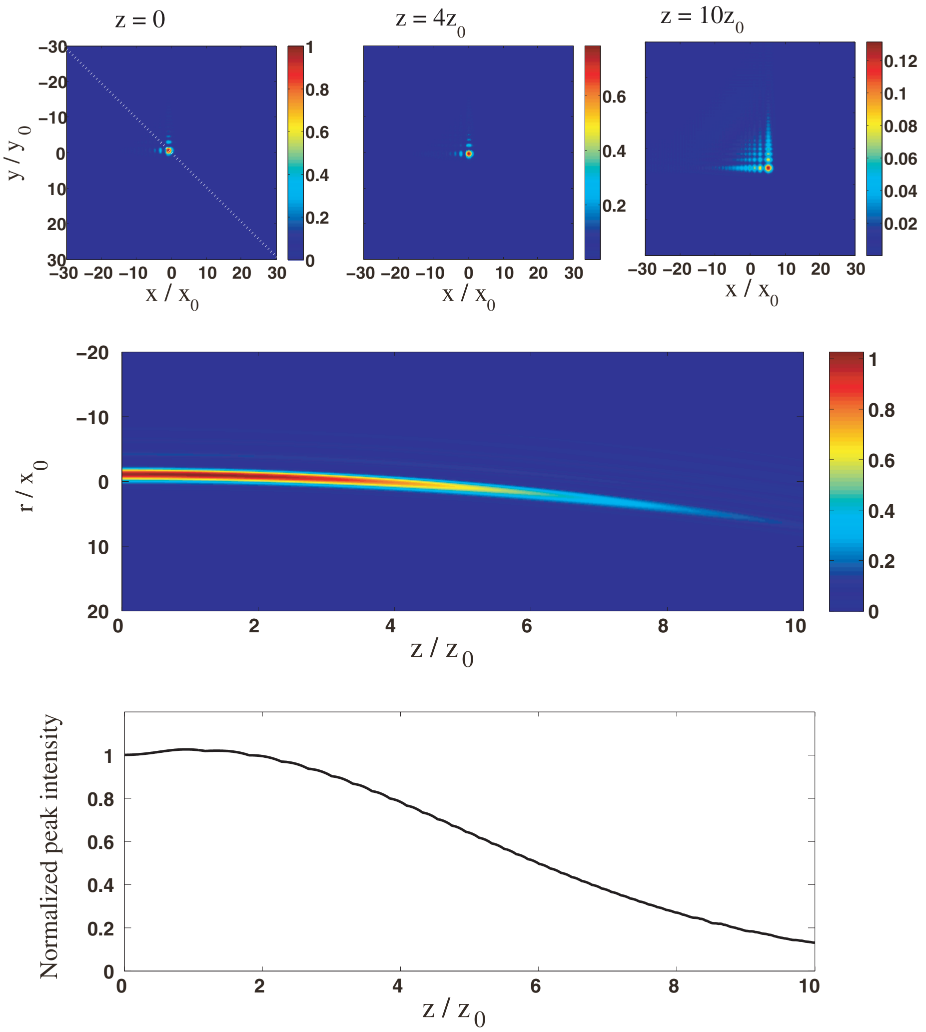

Figure 1. The intensity distribution of an Airy beam ($a_x = a_y = 0.1$) at different propagation distances with initial powers $P_{in} = 0.01 P_{cr} = 0.2P^G_{cr}$ in a saturated medium. The dashed line in the first plot indicates the position of the longitudinal cross section. The middle plot indicates the evolution of intensity distribution in the longitudinal cross section during propagation and the bottom plot shows the normalized peak intensity as a function of the propagation distance.

different propagation distances are shown in figures 1–4. All intensity distributions have been normalized to their initial peak intensity. Diffraction and the spatially variant nonlinear refractive index resulted from the third-order self-focusing effect and the fifth-order self-defocusing effect, leading to a redistribution of the intensity in the Airy beam. In order to obtain a good understanding of the collapsing beams in the Airy beam, we simulate the nonlinear propagation of the Airy beam in a saturated medium with three different initial powers: (i) $P_{in} << P_{cr}$, (ii) $P_{in} \gtrsim P_{cr}$ and (iii) $P_{in} >> P_{cr}$. The intensity redistribution in the Airy beam and the appearance and evolution of the collapsing beams are studied and discussed. We first look at the case (i) with initial powers $P_{in} = 0.01P_{cr} = 0.2P^G_{cr}$ ($P_{in} << P_{cr}$). When the initial power P_{in} is much lower than the collapse power P_{cr} of an Airy beam, the propagation behaviour of an Airy beam in a Kerr medium is similar to that in a free space, as shown in figure 1. This is expected because the nonlinear behaviour of the Airy beam (such as the third-order self-focusing nonlinearity and the fifth-order defocusing nonlinearity) is very weak at low input power. In this case, the saturation term is negligible compared to third-order Kerr nonlinearity due to the low input power (i.e. the fifth-order defocusing nonlinear term can be ignored as recognized in equation (1)).

As initial power increases, more energy accumulates in the beam centre due to the self-focusing effect. Upon further increases of initial power, the saturation term (i.e. the fifth order defocusing nonlinearity in equation (1)) begins to play a defocusing role to resist further focusing. The peak intensity in the beam centre oscillates during propagation. Figures 2(*a*) and (*b*) show the propagation of an Airy beam with $P_{in} = 2.5P_{cr} = 50P_{cr}^{G}$ and $P_{in} = 5P_{cr} = 100P_{cr}^{G}$ in the saturated medium, respectively. From figures 2(*a*) and (*b*) show a collapsing beam occurring in the major lobe of the Airy beam. Several collapsing beams with very short propagation distances also appear at the outermost lobes. These collapsing beams shift laterally as a result of the Airy beam (see figures 2(*a*) and (*b*)). However, the collapsing beam in the major lobe of the Airy beam shifts in the opposite direction to ensure the invariance of beam centroid. This is because the lateral-shift effect of the

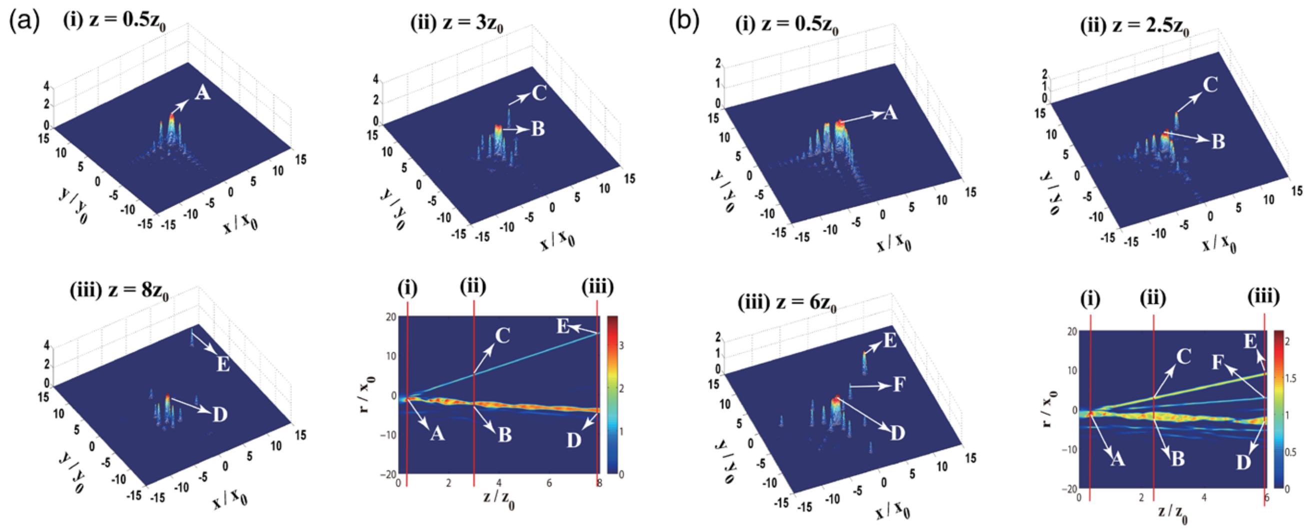

Figure 2. The intensity distribution of an Airy beam in a saturated medium with (*a*) $P_{in} = 2.5P_{cr} = 50P_{cr}^{G}$ and (*b*)$P_{in} = 5P_{cr} = 100P_{cr}^{G}$ for three different propagation distances. The r/x_0 versus z/z_0 plot in (*a*) and (*b*) indicate the evolution of intensity distribution in the longitudinal cross section during propagation. Vertical lines (red) in the r/x_0 versus z/z_0 figures represent the intensity at the propagation distance of (i), (ii) and (iii). The corresponding collapsing beams in the longitudinal cross section are labelled in the figures.

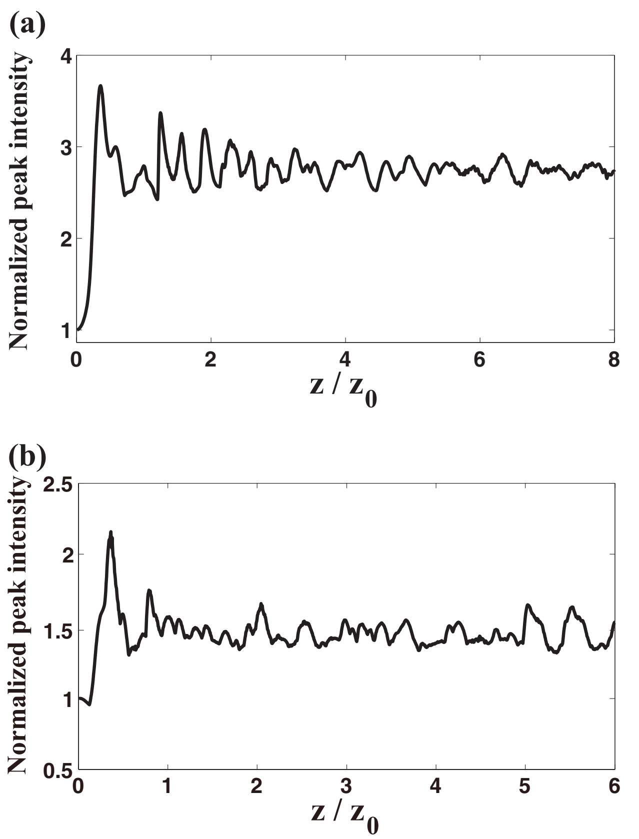

Figure 3. The normalized peak intensity as a function of the propagation distance with different initial powers: (*a*) $P_{in} = 2.5\,P_{cr} = 50\,P_{cr}^{G}$ and (*b*) $P_{in} = 5\,P_{cr} = 100\,P_{cr}^{G}$.

Airy beam on the collapsing beams in the neighbouring lobes is stronger than in the major lobe of the Airy beam located at the beam centre.

Figure 3 shows the normalized peak intensity as a function of the propagation distance with different initial powers: (a) $P_{in} = 2.5P_{cr} = 50P_{cr}^{G}$ and (b) $P_{in} = 5P_{cr} = 100P_{cr}^{G}$. The oscillating normalized peak intensity indicates intensity redistribution and the formation of multiple collapsing beams during propagation. Note here that the peak intensities in figure 3 and figure 4(*d*) have been normalized to their initial peak intensity for convenience of calculations. Here, I_{cr} denotes the initial peak intensity of the Airy beam with initial power $P_{in} = P_{cr}$. In the present work, the initial peak intensities in figure 3(*a*), figure 3(*b*) and figure 4(*d*) are normalized to $2.5I_{cr}$, $5I_{cr}$ and $10I_{cr}$, respectively. It is worthwhile to mention that the peak intensities of the collapsing beams oscillate at almost the same value ~$7.5I_{cr}$, regardless of the initial powers. The results indicate that the oscillations of these collapsing beams in saturated medium are independent of each other. Each collapsing beam exhibits individual collapsing behaviour.

Finally, we consider the case of an Airy beam propagated in a saturated medium with $P_{in} >> P_{cr}$. Figure 4 shows the intensity distribution of an Airy beam in a saturated medium with $P_{in} = 10P_{cr} = 200P_{cr}^{G}$ for different propagation distances. If the initial power is considerably high (i.e. the peak intensity exceeds the saturating intensity in the saturated medium), the peak intensity initially decreases because the fifth-order defocusing effect prevails over the third-order focusing effect under a high input power. More collapsing beams occur in the Airy beam and these collapsing beams also shift laterally as shown in figure 4.

4. Conclusion

In summary, the propagation properties of an Airy beam in a saturated medium are studied both analytically and numerically using a 2D NLS equation with a cubic focusing nonlinearity and a fifth-order defocusing nonlinearity. The analytical description of the total rms beam width of an Airy beam in a saturated medium is obtained. The evolution and appearance of the collapsing beams exhibits novel features

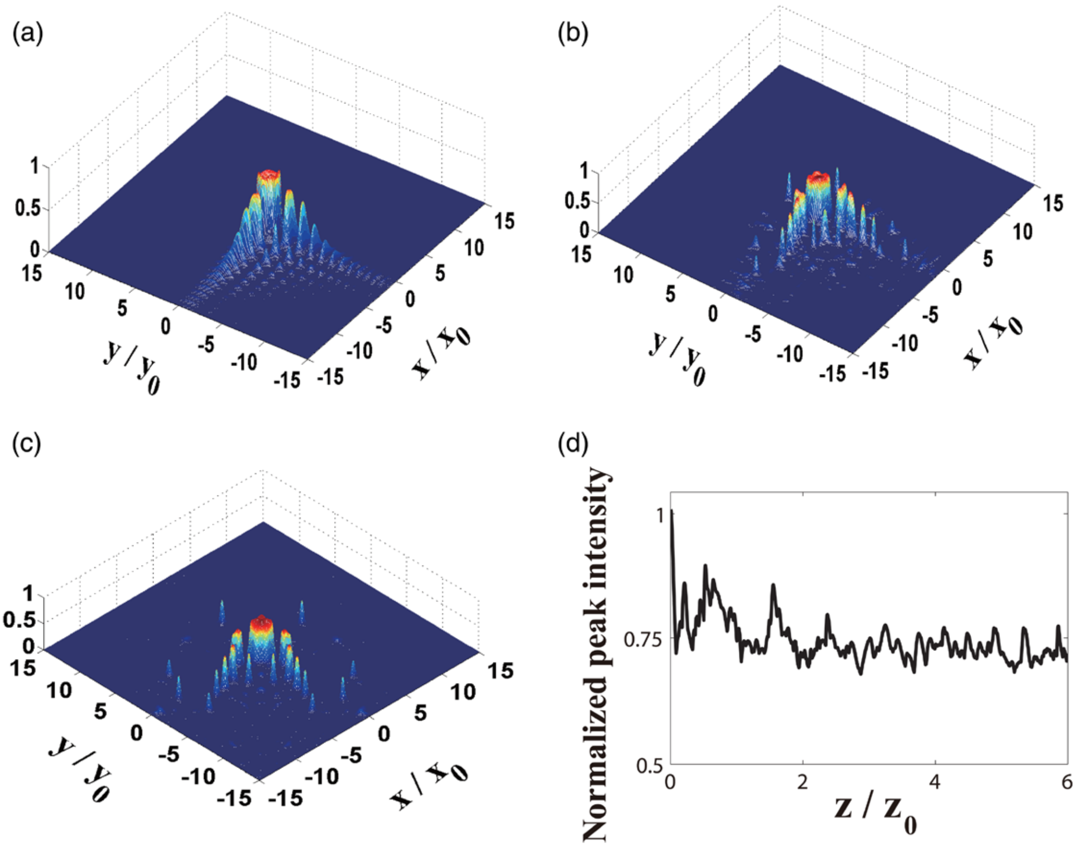

Figure 4. The intensity distribution of an Airy beam in a saturated medium with $P_{in} = 10P_{cr} = 200P_{cr}^{G}$ for different propagation distances (*a*) $z = 0.15z_0$; (*b*) $z = 3z_0$; (*c*) $z = 6z_0$; (*d*) the normalized peak intensity as a function of the propagation distance.

in the saturated medium due to the existence of the third-order focusing effect, the fifth-order defocusing effect, and the effect of the Airy beam. Numerical calculations indicate that the collapsing beams shift laterally due to the effect of the Airy beam whereas the collapsing beam in the major lobe of the Airy beam shifts in the opposite direction for conservation of the centroid. The appearance and evolution of the collapsing beams are closely associated with the initial powers. The peak intensities of the collapsing beams oscillate at almost the same intensity and the intensities are independent of their initial powers. In particular, our study reveals that the location at which the collapsing beam occurs are sensitively dependent on the initial power. Therefore, the initial power is one of the key parameters for manipulating and controlling the collapsing beam and multi-filamentation in an Airy beam. The unique feature of this structured collapsing beam leads to a new phenomenon in light–matter interaction and thus has potential applications in corresponding fields.

Acknowledgments

This work was partially supported by The Science Research Foundation of Zhejiang Sci-Tech University and the National Natural Science Foundation of China (No. 61274017) and the Qianjiang Talent Program of Zhejiang Province (No. QJD1202004).

References

[1] Bergé L 1998 Wave collapse in physics: principles and applications to light and plasma waves *Phys. Rep.* **303** 259–370
[2] Robinson P A 1997 Nonlinear wave collapse and strong turbulence *Rev. Mod. Phys.* **69** 507–73
[3] Bergé L, Gouédard C, Schjødt-Eriksen J and Ward H 2003 Filamentation patterns in Kerr media vs. beam shape robustness, nonlinear saturation and polarization states *Physica* D **176** 181
[4] Vuong L T, Grow T D, Ishaaya A, Gaeta A L, Hooft G W and Eliel E R 2006 Collapse of optical vortices *Phys. Rev. Lett.* **96** 133901
[5] Chen R P, Chew K H and He S 2013 Dynamic control of collapse in a vortex Airy beam *Sci. Rep.* **3** 1406
[6] Polynkin P, Kolesik M, Moloney J, Siviloglou G A and Christodoulides D N 2009 Curved plasma channel generation using ultraintense Airy beams *Science* **324** 229
[7] Polynkin P, Kolesik M and Moloney J 2009 Filamentation of femtosecond laser Airy beams in water *Phys. Rev. Lett.* **103** 123902
[8] Shim B, Schrauth S E, Gaeta A L, Klein M and Fibich G 2012 Loss of phase of collapsing beams *Phys. Rev. Lett.* **108** 043902
[9] Siviloglou G A, Brokly J, Dogariu A and Christodoulides D N 2007 Observation of accelerating Airy beams *Phys. Rev. Lett.* **99** 213901
[10] Kasparian J and Wolf J P 2009 Laser beams take a curve *Science* **324** 194
[11] Broky J, Siviloglou G A, Dogariu A and Christodoulides D N 2008 Self-healing properties of optical Airy beams *Opt. Express* **16** 12880–91

[12] Chu X, Liu Z and Zhou P 2013 Generation of a high-power Airy beam by coherent combining technology *Laser Phys. Lett.* **10** 125102

[13] Baumgartl J, Mazilu M and Dholakia K 2008 Optically mediated particle clearing using Airy wave packets *Nat. Photon*. **2** 675–8

[14] Piksarv P, Valdmann A, Valtna-Lukner H and Saari P 2014 Ultrabroadband Airy light bullets *Laser Phys.* **24** 085301

[15] Ellenbogen T, Voloch-Bloch N, Ganany-Padowicz A and Arie A 2009 Nonlinear generation and manipulation of Airy beams *Nat. Photon.* **3** 395–8

[16] Zhang P, Hu Y, Li T, Cannan D, Yin X, Morandotti R, Chen Z and Zhang X 2012 Nonparaxial Mathieu and Weber accelerating beams *Phys. Rev. Lett.* **109** 193901

[17] Chen R P, Yin C F, Chu X X and Wang H 2010 Effect of Kerr nonlinearity on an Airy beam *Phys. Rev. A* **82** 043832

[18] Makarov V A, Petnikova V M and Shuvalov V V 2013 An Airy beam as a self-similar solution to the problem of slit laser beam propagation in a linear medium and in a photorefractive crystal with diffusion nonlinearity *Quantum Electron*. **43** 931–5

[19] Nithyanandan K, Raja R V J and Porsezian K 2014 Power play in the supercontinuum spectra of saturable nonlinear media *Laser Phys.* **24** 045405

[20] Skarka V, Berezhiani V I and Miklaszewski R 1997 Spatiotemporal soliton propagation in saturating nonlinear optical media *Phys. Rev.* E **56** 1080

[21] Chehrghani A and Torkamany M J 2014 Nonlinear optical properties of laser synthesized Pt nano particles: saturable and reverse saturable absorption *Laser Phys.* **24** 015901

[22] Wang X, Zhou P, Miao Y, Zhang H, Xiao H, Wang X and Liu Z 2014 High power, compact, passively Q-switched Ho-doped fiber laser tandem pumped by a 1150 nm Raman fiber laser *Laser Phys. Lett.* **11** 095101

[23] Malkin V M, 1993 On the analytical theory for stationary self-focusing of radiation *Physica* D **64** 251–66

[24] Pérez-Garćia V M, Torres P and Montesinos G D 2007 The method of moments for nonlinear schrödinger equations: theory and applications *SIAM J. Appl. Math.* **67** 990–15

[25] Weinstein M 1983 Nonlinear schrödinger equations and sharp interpolation estimates *Commun. Math. Phys*. **87** 567–76

[26] Fibich G and Gaeta A L 2000 Critical power for self-focusing in bulk media and in hollow waveguides *Opt. Lett*. **25** 335–7

[27] Press W H, Teukolsky S A, Vetterling W T and Flannery B P 2007 *Numerical Recipes: The Art of Scientific Computing* 3rd edn (Cambridge University Press: New York) chapter 20

[28] Lapidus L and Pinder G F 1982 *Numerical Solution of Partial Differential Equationals in Science and Engineering* (Wiley: New York) chapter 4

[29] Chen R P and Wang H T 2010 Propagation of Laguerre–Gaussian beams in cubic–quintic nonlinear media by variational approach *Opt. Laser Technol*. **42** 1318–22