Influence of $1^+$ states on the $g_R$-factors of ground band in isotopes of $^{160}$Dy and $^{170, 174}$Yb
Influence of $1^+$ states on the $g_R$-factors of ground band in isotopes of $^{160}$Dy and $^{170,174}$Yb

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Abstract

In this work, non-adiabatic effects manifested in the magnetic properties of ground band in even–even deformed nuclei are studied. A simple phenomenological model, which takes into account the Coriolis mixing of states of the ground and $K^π = 1^+_1$ bands, is proposed. Using perturbation theory, corrections to the wave functions of states are determined. In addition, an analytical expression to determine the $g_R$-factor of the states of ground rotational band is obtained. The decrease in $g_R$-factor with increasing angular momentum $I$ is discussed. The accuracy of the approximation is verified. The determined values of wave functions of different methods are found to be close to each other.

Keywords: $g_R$– factors, bandhead energy, energy levels, model, even–even, isotopes

(Some figures may appear in colour only in the online journal)

1. Introduction

The study of the spectroscopic characteristics of rotational states of the ground band still occupies a large part of current research in nuclear structure [1]. The experimentally observed non-adiabaticity in the high spin states of the ground band can, theoretically, be explained
as a result of the intersection of the ground band with other bands that have large values of moment inertia. This non-adiabaticity effect is manifested in the moments of inertia and reduced probabilities of E2-transitions between states located near the intersection.

Experimental values of the reduced probability of E2-transitions between rotational states of ground band is well described by the adiabatic theory [2–7]. Therefore, in deformed even–even nuclei, Bohr–Mottelson and Bengtsson–Frauendorf determined the inertial parameters of the core by using low-spin values of the energy of the ground band [8].

Andrews et al [9] have studied the state of the ground band in practice according to the spin $I = 16\hbar$ isotopes of $^{160,170,174}$Dy and $^{170,174}$Yb using $^{85}$Kr ions with energy of 350 MeV for the Coulomb excitation. Their findings reveal that an increase in the angular momentum $I$ will result in a decrease of the $g_R$-factor.

In this paper, we will study the reasons for the reduction in the observed values of the $g_R$-factor with spin. A simple phenomenological model that takes into account the Coriolis mixing of states of the ground and $K^\pi = 1^+\nu$ bands is proposed. With the use of perturbation theory, corrections in the wave functions of the states are determined. An analytical expression for $g_R$-factor for the states of ground rotational band has been obtained.

2. Wave function of excited states

To analyze the properties of states of the ground band in deformed nuclei, the phenomenological model of [4, 5] has been utilized. The basis states of the Hamiltonian include the ground (gr) and all experimentally known $K^\pi = 1^+\nu$ bands.

The wave functions can be written as follows,

$$\Psi_{K,K'}^I = \sum_{\nu} \Psi_{K,K}^I | IMK \rangle + \sum_{\nu} \Psi_{1^+\nu,K}^I | IM1^+_\nu \rangle.$$  

(1)

Here, $\nu$ is the number of $1^+\nu$ states that have been included in the basis states of the Hamiltonian, $\Psi_{K,K'}^I$ are the amplitudes of mixing of the basis states and takes the form

$$\Psi_{K,K'}^I = \frac{\varphi_{K,K}^I}{\sqrt{\sum_{n=1}^{\nu+1} (\varphi_{K,n}^I)^2}}.$$

(2)

where $\varphi_{K,K}^I$ are the first order corrections obtained by the perturbation theory for the states of $K$–band and has the form: for the ground band ($K = gr$)

$$\varphi_{gr,gr} = 1, \quad \varphi_{1^+\nu,gr} = \omega_{rot}(I) \frac{\langle 1^+_\nu | j_x | gr \rangle}{\omega_{1^+\nu}};$$

(3)

and for $K^\pi = 1^+_\nu$ bands

$$\varphi_{1^+\nu,1^+\nu} = 1, \quad \varphi_{gr,1^+\nu} = -\omega_{rot}(I) \frac{\langle 1^+_\nu | j_x | gr \rangle}{\omega_{1^+\nu}}.$$

(4)

Here, $\omega_{1^+\nu}$ is the bandhead energy, $\langle 1^+_\nu | j_x | gr \rangle$ are the matrix elements of the Coriolis mixing of the states ground and $K^\pi = 1^+_\nu$ rotational bands, and $\omega_{rot}(I)$ is the rotational frequency of the core.

3. Electromagnetic characteristics

In this scheme, for the reduced probabilities of M1-transitions from the states of $K$ bands, we have the following formula:

$$B(M1; IK^\pi \rightarrow I' gr) = \frac{3}{4\pi} 0.06 \left( \frac{\omega_{1^+\nu}}{\omega_{rot}(I)} \right)^2 \varphi_{gr,gr} \sum_{\nu} m_{1^+\nu} \Psi_{gr,1^+\nu}^I \Psi_{1^+\nu,gr}^I C_{1^+\nu,1^+\nu}^I.$$

(5)
Here, \( m'_{11} = \langle 1_1 | \hat{m}(M1)|gr \rangle \) are matrix elements between intrinsic wave functions of ground (gr) and \( K' = 1^+ \) bands determined from experimental data, \( C_{11,1-1} \) are the Clebsch–Gordan coefficients.

This formula in the adiabatic approximation for the transition from \( 1^+ \) levels to the ground state yields,

\[
B(M1; 1^+ 1^+ \rightarrow 0^+ gr) = \frac{3}{4\pi} \left( \frac{e\hbar}{2Mc} \right)^2 \cdot 0.02 (m'_{11})^2. \tag{6}
\]

The magnetic moment of the ground band can be written as follows,

\[
\mu_{gr}(I) = gr \cdot I + \frac{\sqrt{3}}{5} \sum_{\nu} m'_{11} \Psi_{gr,gr}^{I} \sqrt{\frac{I}{I+1}}. \tag{7}
\]

By using the analytical expression equation (2) for the wave functions, we are able to obtain the formula for the \( g_{gr} \)-factor of the ground band states:

\[
g_{gr}(I) = g_0(I = 2) + \frac{\sqrt{3}}{5} \cdot \frac{\alpha_{rot}(I) \cdot \sum_{\nu} (j_{c})_{1,gr} m'_{11} \Psi_{gr,gr}^{I}}{1 + \alpha_{rot}^2(I) \cdot \sum_{\nu} (j_{c})_{1,gr} m'_{11} \Psi_{gr,gr}^{I}}. \tag{8}
\]

This formula allows us to determine the model parameters from the experimentally known values of \( g_{gr} \)-factor of ground band. The values \( g_0 \) are taken from the known \( g \)-factors of the \( I^0 = 2^+ \) states.

In our present model, the expression for the reduced probability of \( E2 \)-transitions within the ground band can be depicted as [4, 5]:

\[
B(E2; Igr \rightarrow (I - 2)gr) = \left\{ \frac{5}{16\pi} eQ_0 \left[ \Psi_{gr,gr}^{I} \Psi_{gr,gr}^{I-2} C_{10;0}^{I-20} + \sum_{\nu} \Psi_{1gr,gr}^{I} \Psi_{1gr,gr}^{I-2} C_{10;2}^{I-21} \right] \right. \\
+ \sqrt{2} \cdot \left[ -\Psi_{gr,gr}^{I-2} \sum_{\nu} m_{11} \Psi_{1gr,gr}^{I-1} C_{11;0}^{20} + \Psi_{gr,gr}^{I} \sum_{\nu} m_{11} \Psi_{1gr,gr}^{I} C_{11;20} \right] \}. \tag{9}
\]

Here, \( m_{11} = \langle gr | \hat{m}(E2)|K' \rangle \) are the matrix elements between the intrinsic wave functions of ground (gr) and \( K' = 1^+ \) bands, whose values are determined from experimental data, and \( Q_0 \)– is the intrinsic quadrupole moment of the nucleus being considered.

4. Results

Calculations have been performed for the isotopes \(^{160}\text{Dy}\) and \(^{170,174}\text{Yb}\). In [9], ground band states up to 16\(\hbar\) of these nuclei were investigated by Coulomb excitation using the ions of \(^{86}\text{Kr}\) with 350 MeV. It was shown that \( g_{gr} \)-factors of ground band state were decreased with the increase of angular momentum for all three nuclei.

In this paper, utilizing the experimental data [9] according to equation (8), the parameters \( (j_c)_{1,gr} = \langle 1_1 | j_c | gr \rangle \) and \( m'_{11} = \langle 1_1 | \hat{m}(M1)|gr \rangle \) have been estimated by the least squares method.

The values of parameters \( m'_{11} \) for \(^{160}\text{Dy}\) and \(^{174}\text{Yb}\) are obtained from experimental data \( B^{exp}(M1; 1^+ 1^+ \rightarrow 0^+ gr) \) from [10, 11], by equation (6)

\[
m'_{11} = \sqrt{\frac{4\pi}{3} B^{exp}(M1; 11^+ \rightarrow 0 gr) \cdot 0.02} \cdot \mu_N. \tag{10}
\]

The parameter \( (j_c)_{1,gr} = (j_c)_{1,gr} \) appears as a free parameter of the calculations, which is described from the best agreement of \( g_{gr}^{\text{theory}} \) with \( g_{gr}^{\text{exp}} \).
Table 1. Value of parameters using in calculation.

<table>
<thead>
<tr>
<th>A</th>
<th>$g_0$</th>
<th>$(j_x)_{1,gr}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{160}$Dy</td>
<td>0.193</td>
<td>0.836</td>
</tr>
<tr>
<td>$^{170}$Yb</td>
<td>0.010</td>
<td>0.338</td>
</tr>
<tr>
<td>$^{174}$Yb</td>
<td>0.030</td>
<td>0.558</td>
</tr>
</tbody>
</table>

Table 2. The amplitudes of mixing for ground states in $^{170}$Yb obtained using perturbation theory and numerical method.

<table>
<thead>
<tr>
<th>$I \cdot \hbar$</th>
<th>Perturbation theory</th>
<th>Numerical method</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>$a_{P,gr}^1$: 0.9993, 0.9977</td>
<td>$a_{P,gr}^1$: 0.9993, 0.9977</td>
</tr>
<tr>
<td></td>
<td>$a_{P,gr}^1$: 0.0145, 0.0258</td>
<td>$a_{P,gr}^1$: 0.0145, 0.0258</td>
</tr>
<tr>
<td></td>
<td>$a_{P,gr}^1$: 0.0071, 0.0126</td>
<td>$a_{P,gr}^1$: 0.0071, 0.0126</td>
</tr>
</tbody>
</table>

In the calculations, in order to improve the calculated values of $g_R$-factor with experiment, the parameters $m'_1$ are ranged. The best agreement of the calculated values with experiment data are the values that were obtained at $(j_x)_{1,gr} = 0$, 0.836 for $^{160}$Dy [10], $(j_x)_{1,gr} = 0$, 0.338 for $^{170}$Yb and $(j_x)_{1,gr} = 0$, 0.558 for $^{174}$Yb [11]. In the calculations, the experimentally known states with $K^\pi = 1^+_1$ are taken into account. The obtained values of normalized parameters $g_0$ and $(j_x)_{1,gr}$ are given in table 1.

For a good agreement with experimental data, we have inclusive $m'_1$, and the best qualitative agreement was obtained with value $3 \cdot m'_1$ for $^{160}$Dy and $^{174}$Yb isotopes. For the case $^{170}$Yb, we do not have experimental data for $B^{exp}(M1; 1^+1^+ \to 0^+gr)$ and for this isotope we set $m'_1 = m'_1$, and it is used as a free parameter. The headband energies of the $K^\pi = 1^+_1$ bands are obtained from the experiment in [10, 11]:

$$\omega_{1_1} = E_{1_1}^{exp}(I = 1) - E_{rot}(I = 1),$$

where $E_{rot}(I)$ is the rotational core energy. The rotational frequency of the core $\omega_{rot}(I)$ and $E_{rot}(I)$ are determined by using Harris parametrization [12]. The values of the inertial parameters of the rotational core are taken from [13].

A comparison between the calculated values of $g_R$-factors with the experimental data [9] for $^{160}$Dy and $^{170,174}$Yb is given in figures 1–3. The comparison shows that the calculated values of $g_R$-factor decrease with angular momentum $I$ and this is in good agreement with experimental data. Hence, a decrease in $g_R$-factors is associated with mixing states of ground band and states $1^+_1$ bands, which possess large values of $B(M1)$ on the ground state.

We have to verify the accuracy of our calculations by numerical diagonality of the Hamiltonian of the matrix using model parameters obtained in the description of the $g_R$-factor. Table 2 and table 3 show that the wave functions of the ground-state bands obtained by the perturbation theory and numerical method for $^{170}$Yb and $^{174}$Yb, respectively, are very close.
Figure 1. The spin dependence of the calculated and experimental values [9] of $g_R$-factor of ground band in $^{160}$Dy.

Figure 2. The spin dependence of the calculated and experimental values [9] of $g_R$-factor of ground band in $^{170}$Yb.
Figure 3. The spin dependence of the calculated and experimental values [9] of $g_R$-factor of ground band in $^{174}$Yb.

Figure 4. Comparison of the calculated and experimental values [7] of the reduced probability of $E2$-transitions in $^{170}$Yb.
Figure 5. Comparison of the calculated and experimental values [11] of the reduced probability of E2-transitions in $^{174}$Yb.

Table 3. The amplitudes of mixing for ground states in $^{174}$Yb, which are obtained using perturbation theory and the numerical method.

<table>
<thead>
<tr>
<th>$I \cdot \hbar$</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Perturbation theory</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\omega_{1,gr}^{l}$</td>
<td>0.9986</td>
<td>0.9956</td>
<td>0.9911</td>
<td>0.9857</td>
<td>0.9795</td>
<td>0.9728</td>
</tr>
<tr>
<td>$\omega_{11,gr}^{l}$</td>
<td>0.0217</td>
<td>0.0389</td>
<td>0.0551</td>
<td>0.0699</td>
<td>0.0836</td>
<td>0.0960</td>
</tr>
<tr>
<td>$\omega_{115,gr}^{l}$</td>
<td>0.0095</td>
<td>0.0170</td>
<td>0.0240</td>
<td>0.0305</td>
<td>0.0364</td>
<td>0.0418</td>
</tr>
<tr>
<td><strong>Numerical method</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\omega_{1,gr}^{l}$</td>
<td>0.9986</td>
<td>0.9957</td>
<td>0.9915</td>
<td>0.9865</td>
<td>0.9811</td>
<td>0.9757</td>
</tr>
<tr>
<td>$\omega_{11,gr}^{l}$</td>
<td>0.0216</td>
<td>0.0384</td>
<td>0.0536</td>
<td>0.0670</td>
<td>0.0787</td>
<td>0.0888</td>
</tr>
<tr>
<td>$\omega_{115,gr}^{l}$</td>
<td>0.0094</td>
<td>0.0169</td>
<td>0.0237</td>
<td>0.0299</td>
<td>0.0355</td>
<td>0.0405</td>
</tr>
</tbody>
</table>

In addition, only the first and last components $\Psi_{1,gr}^{l}$ are provided. The values of the other components $\Psi_{1,gr}^{l}$ can be identified as follows

$$\Psi_{1,gr}^{l} = \Psi_{1,gr}^{l} \omega_{11}^{l}. $$

Determined values associated with the wave functions are found to be close to each other, even for high spins. The calculated values of the probabilities of E2-transitions by equation (9) are shown in figure 4 and figure 5 for the isotopes $^{170,174}$Yb. They are compared with experiments [7, 11]. We do not display the adiabatic values of the reduced probability of E2-transitions in the figures because it carries values that are close to equation (9).
5. Conclusions

In this work, non-adiabatic effects manifested in the magnetic properties of a ground band in even–even deformed nuclei are studied. A simple phenomenological model taking into account the Coriolis mixing of states of the ground and $K^\pi = 1^+$ bands is proposed.

Using perturbation theory, the corrections for the wave functions of states are determined. This paper obtains an analytical expression to determine the $g_R$-factor of the states in the ground rotational band, which allows the model parameters to be determined in collaboration with the experimentally known $g_R$-factor.

The calculations are carried out for isotopes $^{160}$Dy and $^{170,174}$Yb. Finally, it is noteworthy that there is an obvious inverse relation between $g_R$-factor and angular momentum $I$ of the ground band states. This has been explained by the mixture states of the ground and $K^\pi = 1^+$ bands, which have a strong reduced probability of $M1$ to the ground state.

The reduced probabilities of $E2$-transitions within the ground band are calculated, which provides favored correspondence with the experimental data. It is shown that the magnetic characteristics of the ground band of $^{160}$Dy and $^{170,174}$Yb have been found to be more sensitive than that of the electric properties.

Acknowledgments

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